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UNITED STATES AIR FORCE

AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

GA/MC/76D-6

(6) MAXIMUM PAYLOAD, FOUR-IMPULSE, NON-COPLANAR, ORBITAL TRANSFERS FOR AN UPPER STAGE VEHICLE OF THE SPACE TRANSPORTATION SYSTEM.

(9) Master's THESIS,

(14) GA/MC/76D-6

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(12) 83p.

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MAXIMUM PAYLOAD, FOUR-IMPULSE, NON-COPLANAR, ORBITAL TRANSFERS FOR AN UPPER STAGE VEHICLE OF THE SPACE TRANSPORTATION SYSTEM

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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Graduate Astronautical Engineering

December, 1976

Approved for public release; distribution unlimited.

Preface

The subject of this thesis was chosen out of an interest in the United States Air Force space effort. It was my desire to learn more about the Space Transportation System and to investigate the subject of trajectory optimization as it applied to payload maximization. The ultimate objective was to analyze an upper stage vehicle to determine its operational limitations. This thesis is a modification of a number of previous theses and an application of principles that were motivated by the work of Escobal (Ref 4). Basine (Ref 11), Saxon (Ref 12), Tubbs (Ref 9), and Rapp (Ref 13) completed work dealing with the time optimality of both impulsive and finite burn orbital transfers.

I would like to acknowledge my thesis advisor, Major Gerald M. Anderson, for the help and guidance he extended me. I would also like to recognize the loving support and encouragement I received from my wife, Linda, and my two sons, Jeffrey and Gregory, throughout the duration of this project.

Rodney A. Connell

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Notation

English Symbols

A	Inclination Angle Between Orbit Planes
e	Eccentricity
f	True Anomoly
i	Plane Change Angle
I_T	Total Impulse Available
ℓ	Semi-latus Rectum
m	Mass
R	Radius
t	Time
U	Radial Unit Vector
V	Transverse Unit Vector
v	Velocity
W	Transverse Unit Vector
y	Root of Quartic Equation

Greek Symbols

Δ	Increment
θ	Angular Position, Transfer Angle
μ	Gravitational Constant
ϕ	Angular Position
ψ	Angular Position

Superscripts

$\hat{}$	Unit Vector
$\bar{}$	Vector

Notation

Subscripts

1	First Impulse Conditions
2	Second Impulse Conditions
3	Third Impulse Conditions
4	Fourth Impulse Conditions
T	Conditions on Transfer Orbit
i	Variable of Summation

Abstract

Payload capabilities were calculated for an expendable upper stage vehicle compatible with the Space Shuttle Vehicle. Analysis was performed for a four-stage vehicle that was modeled with impulsive thrust and transfer trajectories which obey restricted two-body equations of motion.

The magnitude of the maximum payload deployed into one of two specified orbits when the other payload is known is solved by breaking the four-impulse transfer into two dual-impulse transfer trajectories. The maximum payload solution for one transfer depends upon the specified payload of the other transfer. Each of the dual-impulse transfer trajectories is determined by solving a quartic equation in the square root of the semi-latus rectum of the transfer orbit. Maximum payload capability was dependent upon the available impulse, the angle between terminal orbit planes, the difference in the radii of the terminal orbits, the plane changes at departure and arrival points, and the transfer angle. Transfer solutions were programmed on a CDC 6600 digital computer.

Computed results indicate that the model vehicle is capable of many non-coplanar orbit-to-orbit transfers that still yield practical payloads. As the transfer angle deviates from the neighborhood around 180^o and the other geometrical parameters increase, the payload decreases.

MAXIMUM PAYLOAD, FOUR-IMPULSE, NON-COPLANAR, ORBITAL TRANSFERS FOR AN UPPER STAGE VEHICLE OF THE SPACE TRANSPORTATION SYSTEM

I. Introduction

Background

The Space Transportation System is composed of a Space Shuttle Vehicle, a propulsive upper stage, and numerous ground systems necessary for support functions. The Space Shuttle Vehicle, illustrated in Fig. 1, consists of a payload-carrying Orbiter and three external propellant tanks. The Space Shuttle Vehicle will take off vertically from a launch pad, powered by the three main engines of the Orbiter and the two solid rocket boosters. The solid rocket boosters are jettisoned after their expiration. Still powered by its main engines, the Orbiter continues ascent into earth orbit. The external tank is jettisoned just prior to orbital insertion (Ref 2:419). Payloads of up to 65,000 pounds can be carried in the cargo bay, which is 15 feet in diameter by 60 feet in length. The onboard maneuvering system of the Orbiter makes it possible to climb to an orbital altitude of up to 250 nautical miles (Ref 1:2.5.1).

The propulsive upper stage will be carried into low earth orbit in the paylaod bay of the Orbiter. Once on station, the bay doors will open and the upper stage vehicle will be positioned for launch (see Fig. 2). DOD has selected Burner II, an expendable interim upper stage (IUS) vehicle, to perform shuttle missions until approximately 1984, at which time NASA plans completion of a fully reusable upper

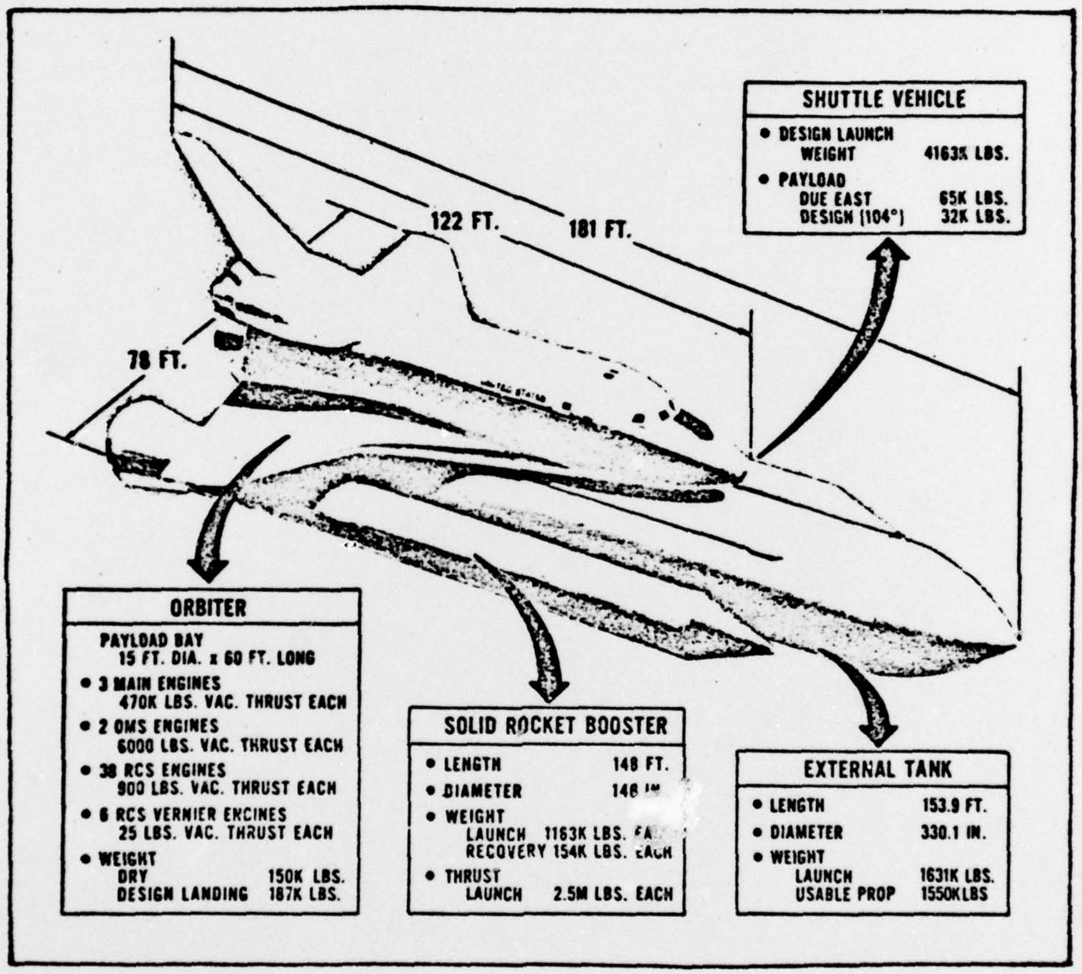


Fig. 1. Space Shuttle Vehicle
(From Ref 14:35)

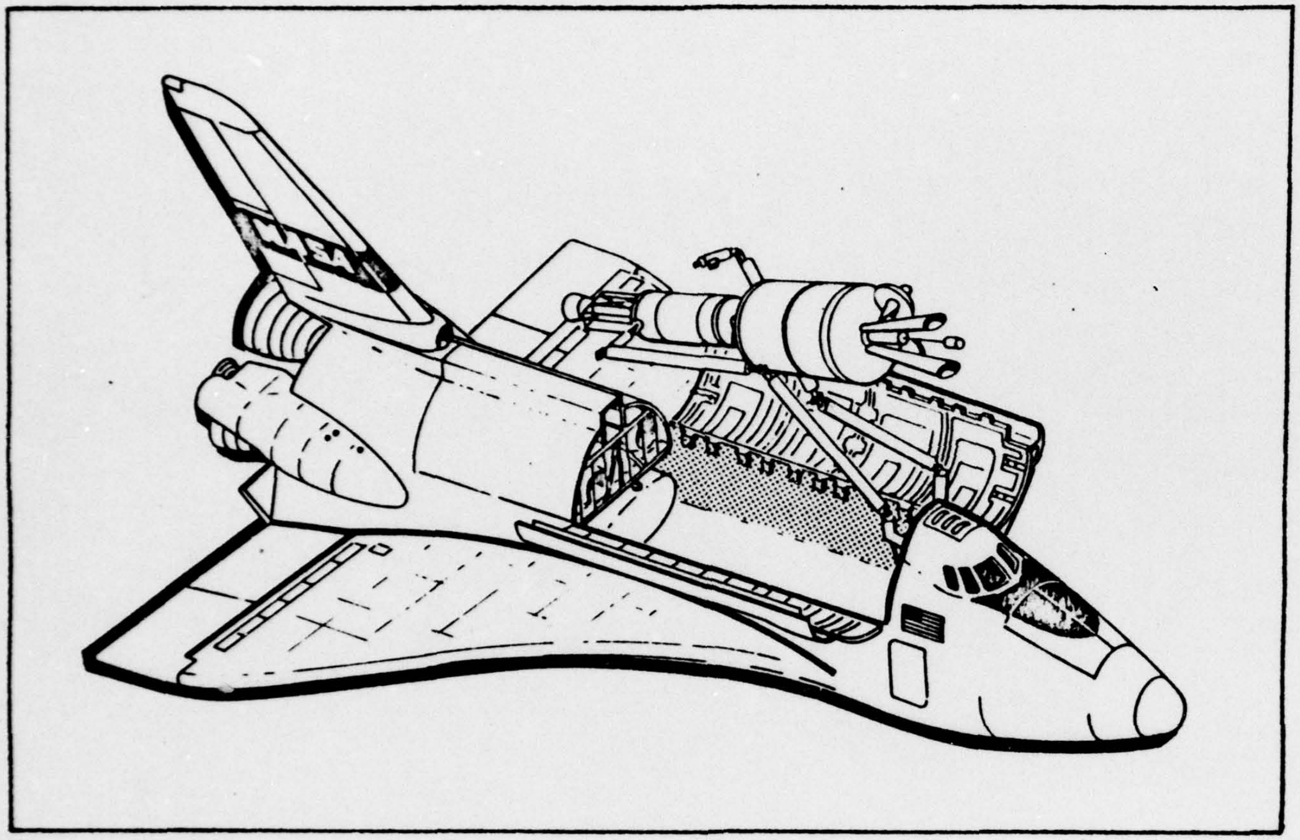


Fig. 2. Positioning Upper Stage for Launch
(From Ref 2:25)

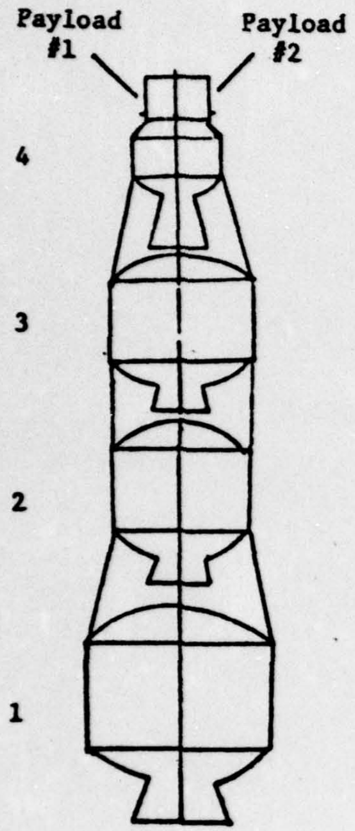
stage vehicle.

During a typical mission, the Space Shuttle Vehicle delivers the propulsive upper stage and satellite into orbit around the earth. The Space Shuttle Vehicle releases the upper stage and satellite, as a unit, while in this orbit. The propulsive upper stage then expends its energy to maneuver the satellite into a different orbit around the earth. Eventually, the Space Transportation System will allow for placement, retrieval, and repair of orbiting satellite systems anywhere around the earth (Ref 7:14-17).

Using Burner II design philosophy, the Boeing Aerospace Company, Space Systems Division, has developed a family of solid rocket motor stages, based upon a two-stage model, that meets the requirements for a low-cost, expendable interim upper stage. Since the Burner II two-stage vehicle model is a short, compact unit, two will fit in the Orbiter payload bay. With a four-stage tandem vehicle configuration, it is possible to deploy two satellites and only use one IUS vehicle. As a result, the shuttle flights needed to launch spacecraft that use the IUS can be greatly reduced. For this reason, this thesis analyzes a four-stage tandem vehicle configuration. Details of the vehicle are shown in Fig. 3. The weights and propulsion summary of the four-stage tandem vehicle configuration under analysis appear in Table I.

Statement of the Problem

The problem is to calculate the maximum transferrable payload capability of a four-stage solid rocket vehicle that is capable of deploying two payload packages into two different, non-coplanar orbits around the earth. The mass of one payload will be specified so that with the given fixed-fuel loading the unknown payload can be maximized



MOTOR NO.	AVERAGE THRUST	AVERAGE BURN TIME	SPECIFIC IMPULSE
1	40,000 LB	135 SEC	300 SEC
2	40,000 LB	100 SEC	285 SEC
3	29,000 LB	100 SEC	290 SEC
4	14,700 LB	80 SEC	250 SEC

Fig. 3. Four-Stage Vehicle Model

Table I. Model Vehicle Weights and Propulsion Summary

Total Orbiter Payload Capacity	65,000 LB
Total IUS Weight	54,700 LB
Stage 1	
Stage Weight	20,400 LB
Propellant Weight	18,000 LB
Total Inert Weight	2,400 LB
Average Thrust	40,000 LB
Average Burn Time	135 SEC
Stage 2	
Stage Weight	16,200 LB
Propellant Weight	14,000 LB
Total Inert Weight	2,200 LB
Average Thrust	40,000 LB
Average Burn Time	100 SEC
Stage 3	
Stage Weight	12,000 LB
Propellant Weight	10,000 LB
Total Inert Weight	2,000 LB
Average Thrust	29,000 LB
Average Burn Time	100 SEC
Stage 4	
Stage Weight	6,100 LB
Propellant Weight	4,700 LB
Total Inert Weight	1,400 LB
Average Thrust	14,700 LB
Average Burn Time	80 SEC

with respect to the following parameters:

1. The total impulse available,

2. The angle between the planes in which the terminal orbits lie,

3. The orbital radius,

4. The inclination angles required to transfer from the initial orbit into the transfer orbit and from the transfer orbit into the target orbit, and

5. The transfer angle between the point of departure and the point of arrival as measured between the radius vectors.

Only trajectories corresponding to the shortest transfer arc are considered because of their faster transfer times and usefulness in orbital rendezvous missions. The satellite or other payload weight which is delivered by the propulsive upper stage can vary for any particular mission as long as the combined upper stage plus payload does not exceed 65,000 pounds.

The purpose of this study is to investigate the maximum payload capability for a proposed tandem configuration of a propulsive upper stage. Specifications of the vehicle being analyzed resemble the Boeing Company's proposal that resulted in the selection of Burner II for the IUS (Ref 10). The fundamental assumption was that the vehicle employed impulsive thrust. Other assumptions include the following:

1. The vehicle remains in orbits about a spherical earth,

2. The vehicle is free from perturbations due to other masses,

3. The vehicle is free from drag,

4. The vehicle is moving under the influence of an inverse square gravitational field,

5. No gravitational losses occur, and

6. All transfers are non-coplanar.

General Approach

The approach taken to solve the orbit-to-orbit transfer problem was to determine the maximum payload mass for which a transfer trajectory existed between points in two terminal orbits. The object was then to define the boundaries of the vehicle's payload capability by extending each parameter of the problem.

The four-impulse trajectory profile is a discontinuous set of two dual-impulse transfers from a known point in the initial orbit to a specified point in the target orbit. An additional constraint imposed upon the problem is that each dual-impulse trajectory use all of the total impulse available from the two solid rocket motors required to make the transfer.

Each dual-impulse solution is in the form of a quartic equation in the square root of the semi-latus rectum of their respective transfer orbit. The coefficients of the quartic equation are functions of the geometry of the transfer, the total impulse available, and the velocity components required to maintain the specified terminal orbits.

Upon factoring the quartic equation, candidate values for the semi-latus rectum of the transfer trajectory result. Maximum payload is defined as that payload mass just before the condition that no candidate values for the transfer trajectory exist. No candidate values corresponds to the point where the total system mass is increased until all roots of the quartic equation become complex.

Options for fixing the mass of the first payload deployed or fixing the mass of the second payload deployed are incorporated into the computer program. The maximum payload solution then depends upon the value of the mass specified for one of the payloads. The

major resource required for this thesis is the CDC 6600 digital computer.

Sequence of Presentation

The results of this thesis are presented as follows: In Chapter II, the two-impulse transfer is analyzed. A summary of the development of the two-impulse solution is presented. Geometry of the non-coplanar transfer is defined. In Chapter III, the results of this thesis are presented. Some operational limitations for the model analyzed are defined. The thesis is concluded with a summary of conclusions and recommendations for further study in Chapter IV.

II. Calculation of Impulsive Transfers

Impulsive Thrust

One advantage of the impulsive thrust approximation is that it simplifies what might otherwise have been a complicated analysis. However, a disadvantage is that it is not extremely accurate. For an impulsive thrust, the magnitude of the thrust is assumed to be infinite and applied over an infinitesimally small time period. The impulsive thrust assumption, therefore, is only an approximation of the more realistic program of finite thrust. However, application of the calculus of variations method to optimize finite thrust trajectories generally results in a nonlinear two-point boundary value problem. Closed form solutions of this nonlinear two-point boundary value problem are generally not available. The numerical techniques used to solve the two-point boundary value problem often require considerable computer resources, time, and expense. The effect of an impulse is an instantaneous change in velocity without a corresponding change in position. The entire transfer trajectory is a conic section which obeys the restricted two-body equations of motion.

In Reference 13, Rapp has shown that the impulsive solution is a good first approximation to the finite thrust solution. In Reference 8, Robbins presents formulas for estimating the performance penalties arising from the use of impulses instead of finite thrust for orbit transfer maneuvers. Finally, in Reference 5, Handelsman demonstrates how guessing the Lagrangian multiplier functions $\lambda_i(t)$ may be eliminated by use of the $\lambda_i(0)$ from impulsive trajectories

to initiate iteration for nonimpulsive fixed-thrust, fixed-exhaust velocity propulsion. The impulsive approximation becomes less severe as the value of the thrust increases and the thrusting periods decrease with respect to the coasting period along a thrust-coast-thrust trajectory profile.

Maximum Payload Problem

The maximum payload is constrained by the total amount of fuel available for the transfer. The use or ejection of fuel mass, dm, results in an increase of velocity, dv. If the velocity and initial mass of the propulsive upper stage at some time t is given as v_o and m_o, respectively, and the exhaust velocity (measured with respect to the IUS vehicle) is given as v_e, then the expression for the change in velocity in an inverse square gravitational field is

$$\Delta V = V - v_o = -v_e \ln\left(m_o/m\right) - \int_0^t g(r)\,dt \tag{1}$$

If the exhaust velocity is defined as

$$v_e = -\frac{\text{thrust}}{\Delta m/\Delta t} \tag{2}$$

and gravity losses are assumed negligible, then the change in velocity due to an impulsive thrust becomes

$$\Delta V = \frac{\text{thrust}}{\Delta m/\Delta t} \ln\left(m_o/m\right) \tag{3}$$

If, now, a constraint is imposed which states that all the available

fuel be used in the performance of each orbital transfer, then the total change in velocity or impulse becomes

$$\sum \Delta V_i = I_T \tag{4}$$

where ΔV_i = the individual impulses used for transfer and

I_T = the total impulse available.

Analytical expressions for the calculation of the change in velocity at each of the four impulsive thrusts appear in Appendix A.

Coordinate Frame

In order to generate a solution for the four-impulse problem, a suitable coordinate system must first be selected to describe the initial and final position and velocity components at the specified end points. In this non-coplanar case, a rotating spherical coordinate system, illustrated in Fig. 4, was used. The symbol $\hat{U}$ is a unit vector in the radial direction, parallel to radius vector, $\overline{R}$. The symbol $\hat{V}$ is a tangential unit vector normal to $\hat{U}$ and in the direction of the orbital motion. The symbol $\hat{W}$ is the out of the plane component of the mutually orthogonal set of unit vectors that is defined by the cross product of $\hat{U}$ and $\hat{V}$.

Geometry of Impulsive Transfer

The geometry for the non-coplanar two-impulse transfer is illustrated in Fig. 5. The vehicle is orbiting the earth in an initial parking orbit, as designated by orbit 1. There is one instantaneous thrust from the initial orbit 1 into the transfer orbit, occurring at the designated initial point. The change in velocity at this point is given by.

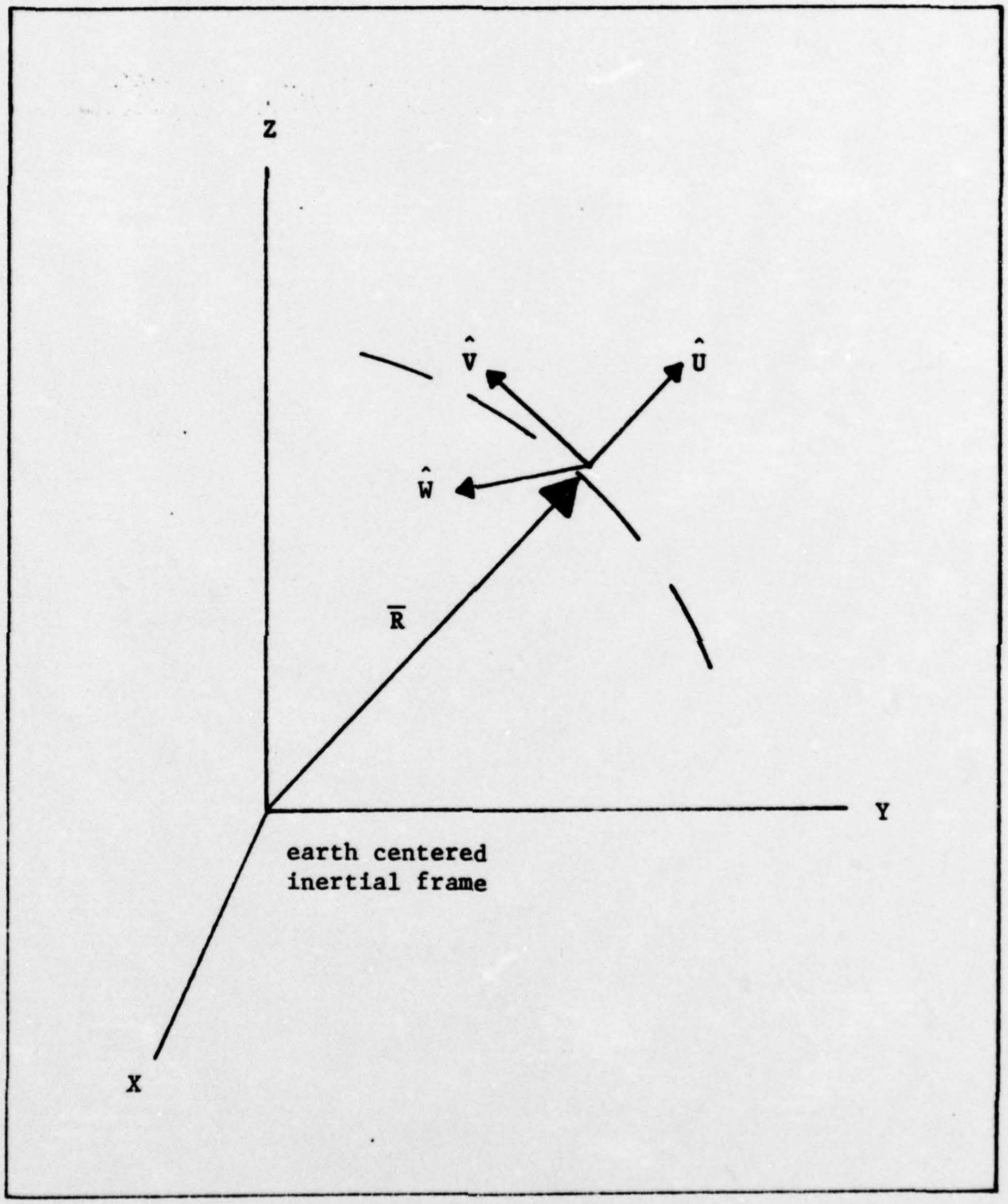


Fig. 4. Rotating Spherical Coordinate System

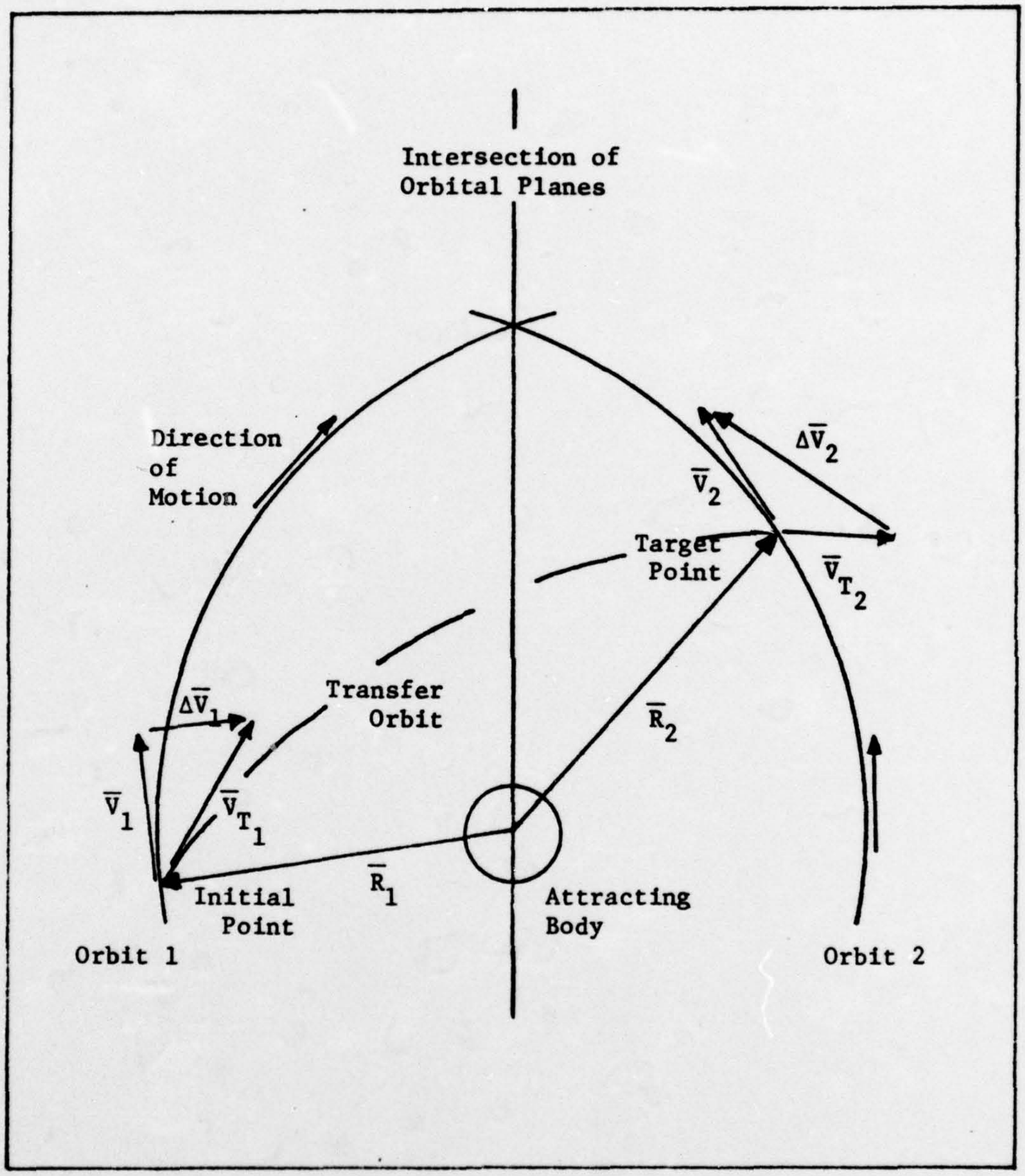


Fig. 5. Geometry of Impulsive Transfer

$$\Delta\overline{V}_1 = \overline{V}_{T_1} - \overline{V}_1 \tag{5}$$

There is a second instantaneous thrust required to transfer into the final orbit 2, occurring at the designated target point. The change in velocity at this point is given by

$$\Delta\overline{V}_2 = \overline{V}_2 - \overline{V}_{T_2} \tag{6}$$

The third instantaneous thrust is identical to the first thrust, except it transfers the vehicle from orbit 2 into the second transfer orbit. The fourth instantaneous thrust is identical to the second thrust, except it transfers the vehicle into the final orbit 3. The expressions for the third and fourth impulses are

$$\Delta\overline{V}_3 = \overline{V}_{T_2} - \overline{V}_2 \tag{7}$$

and

$$\Delta\overline{V}_4 = \overline{V}_3 - \overline{V}_{T_2} \tag{8}$$

respectively. The four-impulse trajectory profile is a combination of two dual-impulse trajectory profiles. For example, the two-impulse trajectory profile is a direct transfer from a known point, designated by $\overline{R}_1$, to a specified point in the target orbit, designated by $\overline{R}_2$. Another direct transfer occurs from orbit 2 to orbit 3. The vehicle must deploy the first payload into orbit 2, so this intermediate orbit is also the orbit in which any necessary phasing is accomplished between maximum payload transfer trajectories.

This would seem to be a relatively simple problem. However, when dealing with non-coplanar orbits and when not transferring from an initial position along the intersection of the orbit planes, each target position dictates a different total amount of plane change. This plane change represents a use of the total impulse available.

Geometry of Non-Coplanar Transfer

Other geometry for the non-coplanar, orbit-to-orbit transfer is illustrated in Fig. 6. The arrows designate the direction of orbital motion. The parameters pictured and investigated in this study are defined as follows:

1. A, the angle between the planes in which the terminal orbits lie,
2. R, the radius to the target orbit,
3. i_1, the inclination angle measured from the initial orbit to the transfer orbit,
4. i_2, the inclination angle measured from the transfer orbit to the final orbit, and
5. θ, the transfer angle between the point of departure and point of arrival.

The symbols ϕ and ψ are used to define the initial position and target position, respectively. The initial and target positions are referenced to the line of intersection of the orbit planes and are positive in a direction opposing the direction of the orbital motion. The definitions of geometrical parameters in this problem are important in the development which follows in the next section.

Development of a Two-Impulse Solution

The solution of the four-impulse, maximum payload problem involves the determination of two transfer conics which satisfy the position and velocity boundary conditions of the terminal orbits and maximize

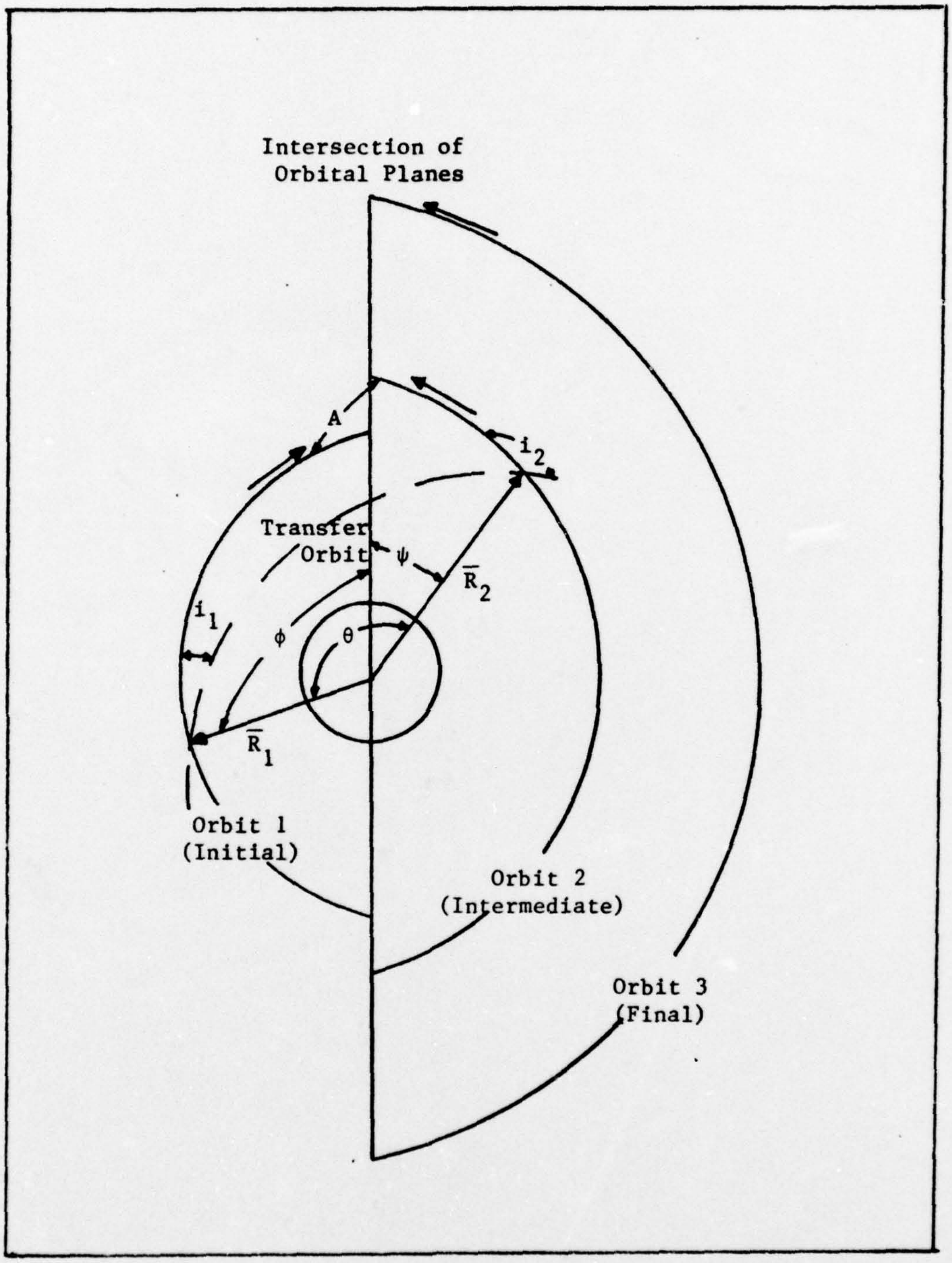


Fig. 6. Geometry of Non-Coplanar Transfer

the payload for each dual-impulse transfer. The two-impulse transfer conic solution follows the same development as that used by Saxon (Ref 12). Saxon used the work of Escobal (Ref 4) as a guideline in his development. The solution to the two-impulse transfer conic is in the form of a fourth order polynomial in the square root of the semi-latus rectum of the transfer orbit. A summary of the development of the polynomial is given in this section. A complete development of the polynomial may be found in Appendix A of Reference 12.

The derivation of the polynomial is based on relating the transfer velocity vectors at the initial and target points to the semi-latus rectum of the transfer orbit while meeting end conditions with a given change in velocity. Using the coordinate system and geometry defined in the previous sections, the position vectors for the initial and target points are

$$\overline{R}_1 = R_1\hat{U}_1 \tag{9}$$

$$\overline{R}_2 = R_2\hat{U}_2 \tag{10}$$

$$\overline{R}_3 = R_3\hat{U}_3 \tag{11}$$

Similarly, the velocity vectors for the initial and target points are

$$\overline{V}_1 = V_{x_1}\hat{U}_1 + V_{y_1}\hat{V}_1 \tag{12}$$

$$\overline{V}_2 = V_{x_2}\hat{U}_2 + V_{y_2}\hat{V}_2 \tag{13}$$

$$\overline{V}_3 = V_{x_3}\hat{U}_3 + V_{y_3}\hat{V}_3 \tag{14}$$

In the formulation of this problem, the eccentricity and distance from the surface of the earth to orbit perigee are known. Using basic two-body orbital mechanics, the vector quantities in Eqs. (12), (13), and (14) are calculated directly.

In the plane of the transfer conic, the equation for the transfer velocity vector at the initial point is

$$\overline{V}_{T_1} = \dot{R}_1 \hat{U}_{T_1} + R_1 \dot{f}_{T_1} \hat{V}_{T_1} \tag{15}$$

where $\hat{U}_{T_1}$ = radial unit vector

$\hat{V}_{T_1}$ = tangential unit vector in the direction of orbit motion

R_1 = radius to initial point

f_{T_1} = angle between R_1 and the perigee of the transfer orbit

The general conic equation in polar form yields

$$R_1 = \frac{\ell_T}{1 + e_T \cos f_{T_1}} \tag{16}$$

and

$$R_2 = \frac{\ell_T}{1 + e_T \cos(f_{T_1} + \theta)} \tag{17}$$

where ℓ_T = transfer arc semi-latus rectum

e_T = transfer arc eccentricity

θ = angle between $\overline{R}_1$ and $\overline{R}_2$

R_2 = radius to target point

Solving for ℓ_T from Eqs. (16) and (17); and after some other manipulations we arrive at the tangential velocity component

$$R_1 \dot{f}_{T_1} = \frac{\sqrt{\mu \ell_T}}{R_1} \tag{18}$$

where μ = the gravitational constant.

The radial velocity component is

$$\dot{R}_1 = \frac{\sqrt{\mu/\ell_T}\,(1 - \cos\theta)}{\sin\theta} + \frac{\sqrt{\mu \ell_T}\,(R_2\cos\theta - R_1)}{R_1 R_2 \sin\theta} \tag{19}$$

Substituting the radial and tangential velocity components into Eq. (15) provides the transfer velocity vector, $\overline{V}_{T_1}$, at the initial point as

$$\overline{V}_{T_1} = \left[\frac{\sqrt{\mu}(1 - \cos\theta)\ell_T^{-1/2}}{\sin\theta} + \frac{\sqrt{\mu}(R_2\cos\theta - R_1)\ell_T^{1/2}}{R_1 R_2 \sin\theta}\right] \hat{U}_{T_1} + \left[\frac{\sqrt{\mu}\;\ell_T^{1/2}}{R_1}\right] \hat{V}_{T_1} \tag{20}$$

In a similar manner, the transfer velocity vector

$$\overline{V}_{T_2} = \dot{R}_2 \hat{U}_{T_2} + R_2 \dot{f}_{T_2} \hat{V}_{T_2}$$

takes the following form:

$$\overline{V}_{T_2} = \left[\frac{\sqrt{\mu}(\cos\theta - 1)\ell_T^{-\frac{1}{2}}}{\sin\theta} + \frac{\sqrt{\mu}(R_2 - R_1\cos\theta)\ell_T^{\frac{1}{2}}}{R_1R_2\sin\theta} \right] \hat{U}_{T_2}$$

$$+ \left[\frac{\sqrt{\mu}\ \ell_T^{\frac{1}{2}}}{R_2} \right] \hat{V}_{T_2} \tag{21}$$

Vector addition on the geometry as illustrated in Fig. 5 gives

$$\Delta\overline{V}_1 = \overline{V}_{T_1} - \overline{V}_1 \tag{22}$$

and

$$\Delta\overline{V}_2 = \overline{V}_2 - \overline{V}_{T_2} \tag{23}$$

By definition, since we are restricted to the plane of the transfer conic,

$$\overline{V}_1 = (V_{x_1})\hat{U}_1 + (V_{y_1})\hat{V}_1 \tag{24}$$

and

$$\overline{V}_2 = (V_{x_2})\hat{U}_2 + (V_{y_2})\hat{V}_2 \tag{25}$$

The constraint that the total impulse available be entirely used can be written

$$\left(\Delta V_1^2\right)^{\frac{1}{2}} + \left(\Delta V_2^2\right)^{\frac{1}{2}} = I_T \tag{26}$$

Squaring Eq. (26) twice and simplifying terms yields

$$\left(\Delta V_1^2 - \Delta V_2^2\right)^2 - 2I_T^2\left(\Delta V_1^2 + \Delta V_2^2\right) + I_T^4 = 0 \qquad (27)$$

Expressions for ΔV_1^2 and ΔV_2^2 are obtained by taking the dot products

$$\Delta V_1^2 = \Delta\overline{V}_1 \cdot \Delta\overline{V}_1 \qquad (28)$$

$$\Delta V_2 = \Delta\overline{V}_2 \cdot \Delta\overline{V}_2 \qquad (29)$$

Substituting for ΔV_1^2 and ΔV_2^2 from Eqs. (28) and (29) into Eq. (27), multiplying by ℓ_T, and redefining the coefficients of powers of ℓ_T yields

$$Ay^4 + By^3 + Cy^2 + Dy + E = 0 \qquad (30)$$

where $y = \ell_T^{1/2}$.

The coefficients of Eq. (30) are defined as follows:

$$A = \beta_1^2 - 2I_T^2\beta_4$$

$$B = 2\beta_1\beta_3 - 2I_T^2\beta_5$$

$$C = 2\beta_1\beta_2 + \beta_3^2 - 2I_T^2\beta_8 + I_T^4$$

$$D = 2\beta_2\beta_3 - 2I_T^2\beta_6$$

$$E = \beta_2^2 - 2I_T^2\beta_7$$

Expressions for the β values appear in Appendix B and are functions

of the following:

1. The velocity components necessary to maintain the initial and target orbits,
2. The plane change angles,
3. The transfer angle, and
4. The radius of both the initial and target orbit.

Solutions to the transfer from the initial orbit to the intermediate orbit are computed using one polynomial. Transfer from the intermediate orbit to the final orbit simply requires reimplementation of the polynomial expression whose derivation is summarized above. The fourth order polynomials in the form of Eq. (30) are factored using a FORTRAN extended subroutine called DMULR. DMULR finds all the complex roots of a polynomial with real coefficients and uses double precision arguments. Upon factoring the quartic equation, four values for the semi-latus rectum of candidate trajectories resulted. Normally two values were complex while the other two values were real. If too much impulse was available, then all four roots of Eq. (30) could be real and all four transfer trajectories were possible candidates. If insufficient impulse was available, then all four roots became complex and no candidate was assumed to exist for the transfer trajectory.

Having completed the transfer solution to a given orbit, the total vehicle mass was then increased and the procedure repeated. The maximum payload mass was defined as the mass just before all the roots became complex. This condition corresponded to no possible trajectory for the increased payload mass or, equivalently, no

possible trajectory for the available impulse.

With a value of the square root of the semi-latus rectum, $\ell_T^{1/2}$, of the transfer trajectory, a transfer velocity vector could be evaluated. Knowing the transfer velocity and corresponding position allowed the determination of all the classical elements of the transfer arc by means of the equations of two-body orbital mechanics. If the coplanar case were of interest, the angles i_1 and i_2 , which define the plane changes required at the initial and target points, are set equal to zero.

III. Results

Normalized System of Units

To simplify the computer programming and the input and output data, a standard set of geocentric units were used. The unit of distance, 1 DU, is defined as the earth's mean equatorial radius. The unit of velocity, 1 DU/TU, is defined by the velocity of an object in a circular orbit of radius 1 DU. For this set of units, the value of the gravitational constant, μ, is 1 DU^3/TU^2. The conversion from English units to canonical units are given by

$$1 \text{ n.mi.} = 2.903656 \times 10^{-4} \text{ DU}$$

$$1 \text{ sec} = 1.239444 \times 10^{-3} \text{ TU}$$

$$1 \text{ ft/sec} = 3.855604 \times 10^{-5} \text{ DU/TU}$$

This set of values was extracted from Appendix A of Reference 3.

Comments on the Computer Program

To perform the four-impulse transfer, maximum payload transfers were computed in two steps. The first transfer was from the initial to the intermediate orbit. The second transfer was from the intermediate orbit to the final orbit. The weight of the vehicle before departing from the initial orbit included four solid rocket motors and two separate payloads. At the intermediate orbit, the first payload was deployed. The first and second solid rocket motors were assumed to be jettisoned after their use. The weight of the vehicle before departing from the intermediate orbit, therefore, included the remaining two solid rocket motors plus the remaining

payload.

Between each orbit change there occurs two impulses and two plane changes. The plane change at the initial point of each orbit change was through an angle i_1 and at the final point was through an angle i_2 (see Fig. 6). Because the spherical trigonometric relationships held true only for orbits of the same altitude, departure and arrival points on terminal orbits of different altitude were specified by the transfer angle only.

Data read into the main computer program included:

1. The eccentricities ($e = 0$) of the initial, intermediate, and final orbits,
2. The radial distance above the earth's surface of each orbit,
3. The average thrust for each of the four solid rocket motor stages,
4. The average burn time for each of the four solid rocket motors, and
5. The stage weight (propellant, structure, and motors) for each of the four solid rocket motor stages.

In addition, one payload weight and the orbit in which it was to be deployed was specified.

In subroutine TRAJ, the eccentricity and semi-major axis of the transfer trajectory were computed. Also, the total impulse available was checked for each possible trajectory to insure that impulse constraints were satisfied.

Payload Definition

In this thesis, the payload weight is defined to be the useful payload unloaded at the destination plus any weight due to supporting structures, such as a housing container. One exception is made to

this attempt to standardize the definition of payload throughout the thesis. This exception occurs whenever the solution to the maximum payload is sought for the first orbit transfer from the initial orbit to the intermediate orbit and the payload into the intermediate orbit has been specified. In this case, the payload weight includes the third and fourth stage weights plus the weight of the payload being carried into the final orbit. Consequently, this defines the entire useful payload unloaded at the intermediate destination and, in a way, is consistent with the original definition.

Maximum Payload Capabilities

The figures and tables presented in this section are all obtained using the model vehicle weights and propulsion specifications that are listed in Table I. Results for a vehicle with different weights and propulsion specifications are presented in Appendix C.

Payload values presented in the figures and tables that follow apply to the capability of the four stage IUS after it has been released from the Orbiter. If the IUS and the two payload packages must be delivered by only one Space Shuttle Vehicle, then the total payload values plus IUS weight could realistically not exceed about 62,500 pounds. (A remote manipulator arm system that prepares the IUS for launch from the Orbiter is assumed to weigh 2,500 pounds.) Of course, larger payloads can be realized for some transfers if the IUS and the two payload packages are delivered to the initial orbit by more than one Space Shuttle Vehicle. In the computation of maximum payloads, five parameters were considered because they affect payload.

Angle Between Orbit Planes. The maximum payload trend that results when the angle between the planes of the terminal orbits increases is illustrated in Fig. 7. The maximum deliverable payload into the final orbit decreased steadily as the angle between the orbit planes increased. For example, a payload of 4000 lbs can be transferred to a final orbit within a 55^{o} plane separation of the intermediate orbit. Parameter values for a 10^{o} inclination angle from intermediate orbit to final orbit appear in Table II.

It is of interest at this point to mention the results that have been recorded by Porter (Ref 6:640). Porter concludes that the fixed two-impulse system can deliver the design payload mass to a final orbit within a 30^{o} plane separation of the intermediate orbit.

Radius to Target Orbit. The radius of the intermediate and final orbits were varied. For a range of inclination angles, Fig. 8 demonstrates that payloads, for example, of approximately 4000 lbs can be placed into synchronous orbit. But, as the radius to the target orbit increases, the maximum payload decreases. Also, the maximum payload will decrease as the inclination angles increase. Table III shows a decrease in the maximum payload also occurs when there is a decrease in the transfer angle.

Inclination Angles. The determination of the maximum payload is dependent upon the total plane change required at the terminal points of the initial and target orbits. The relationship between the angles of these plane changes and payload are illustrated in Figs. 9 and 10. Again, payloads of 4000 lbs can be transferred to synchronous orbit if the required plane change, i_1, from the intermediate orbit to

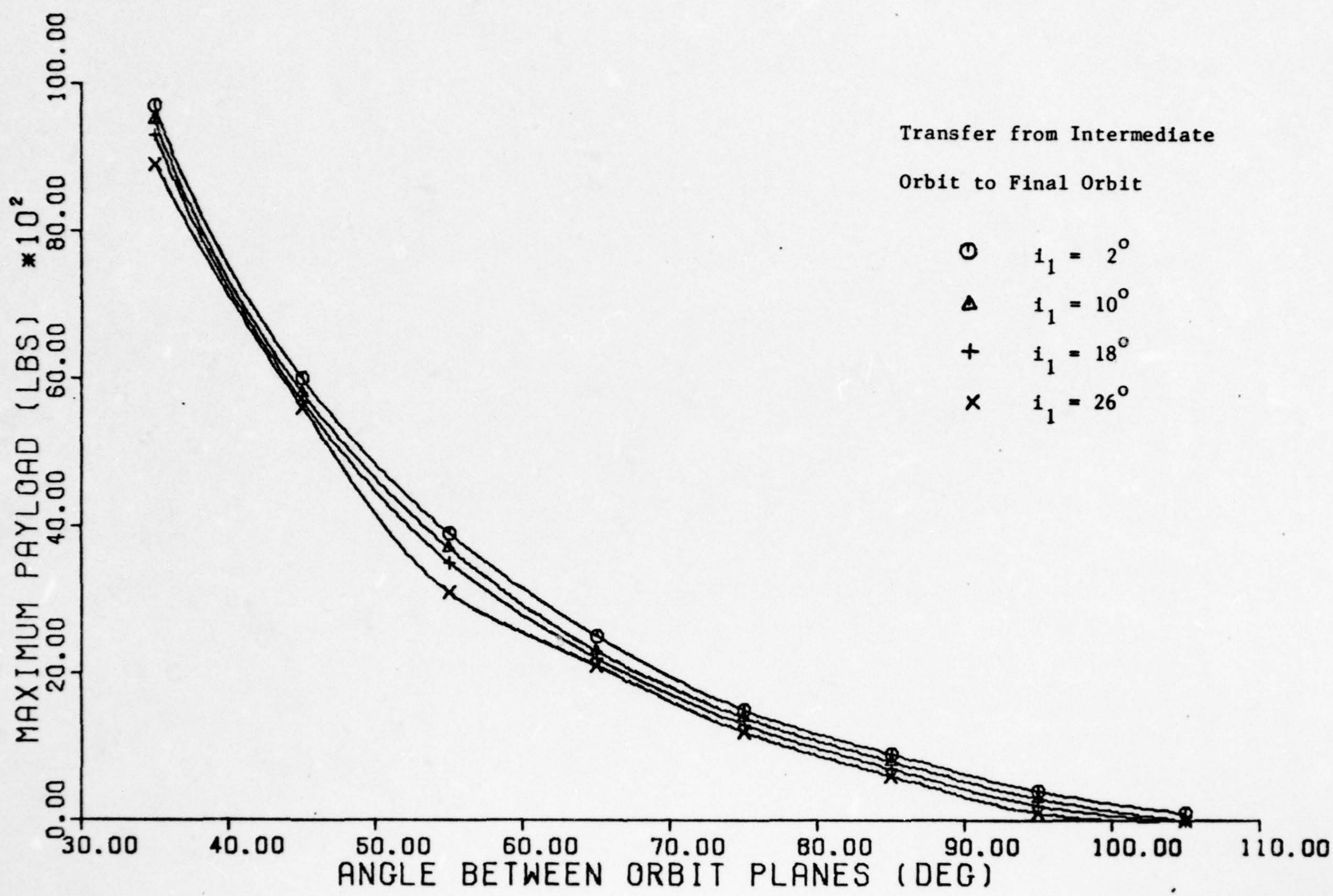


Fig. 7. Maximum Payload vs. Angle Between Orbit Planes
(Payload #1 = 1000 lbs, $R_2 = R_3 = 4.0$ DU, $\phi = 170^o$)

Table II. Maximum Payload vs. Angle Between Orbit Planes
(Paylaod #1 = 1000 lbs, $R_2 = R_3 = 4.0$ DU, $\phi = 170^o$)

	Transfer from Intermediate Orbit to Final Orbit			
Angle Between Orbit Planes (Deg)	$i_1 = 10^o$			
	i_2 (Deg)	θ (Deg)	ψ (Deg)	Payload (LBS)
25	15.2	163.8	6.6	16,300
35	25.2	166.5	4.1	9,500
45	35.2	167.7	3.0	5,800
55	45.2	168.4	2.4	3,700
65	55.2	169.0	2.1	2,300
75	65.2	169.4	1.9	1,400
85	75.2	169.7	1.8	800
95	85.2	170.0	1.7	300
105	95.2	170.5	1.6	0

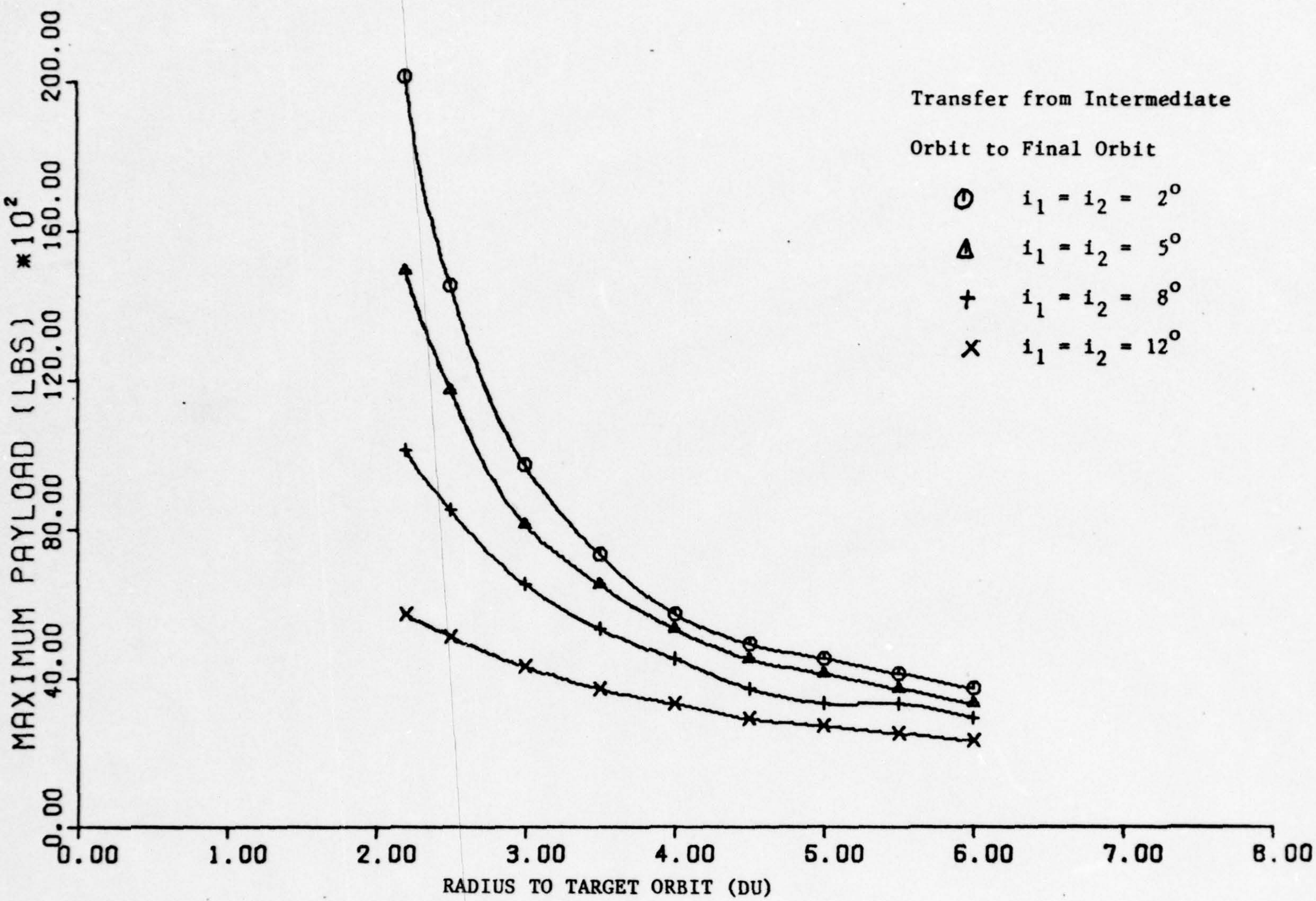


Fig. 8. Maximum Payload vs. Radius to Target Orbit
(Payload #1 = 1000 lbs, $\theta = 150^o$, $R_2 = 1.5$ DU)

Table III. Maximum Payload vs. Radius to Target Orbit
(Payload #1 = 1000 lbs, R_2 = 1.5 DU)

Radius to Target Orbit, R_3 (DU)	Maximum Payload (LBS) for $i_1 = i_2 = 5^o$	
	$\theta = 150^o$	$\theta = 165^o$
2.2	14,900	16,100
2.5	11,700	12,900
3.0	8,100	9,300
3.5	6,500	7,700
4.0	5,300	6,500
4.5	4,500	5,700
5.0	4,100	4,900
5.5	3,700	4,500
6.0	3,300	4,100
Transfer from Intermediate Orbit to Final Orbit		

the plane of the transfer trajectory is within approximately 26°. For a range of values of i_1, Fig. 10, for example, shows that for a plane change, i_2, within approximately 40°, the model vehicle can deliver a 4000 lb payload.

Transfer Angle. The transfer angle between departure and arrival points, as illustrated in Fig. 11, is another factor in the analysis of maximum payload capabilities. As the magnitude of the transfer angle departs from 180°, the payload decreases rapidly. The maximum payload also decreases when the vehicle must transfer between orbits requiring increased plane changes. In addition, Table IV demonstrates that, again, the maximum payload is found to decrease as the radius of the target orbit increases. A 5000 lb payload, for instance, can be delivered to synchronous orbit if the transfer angle is within $\pm 10^{\circ}$ of 180°. But, the maximum payload decreases to 3000 lbs if the transfer angle falls within $\pm 40^{\circ}$.

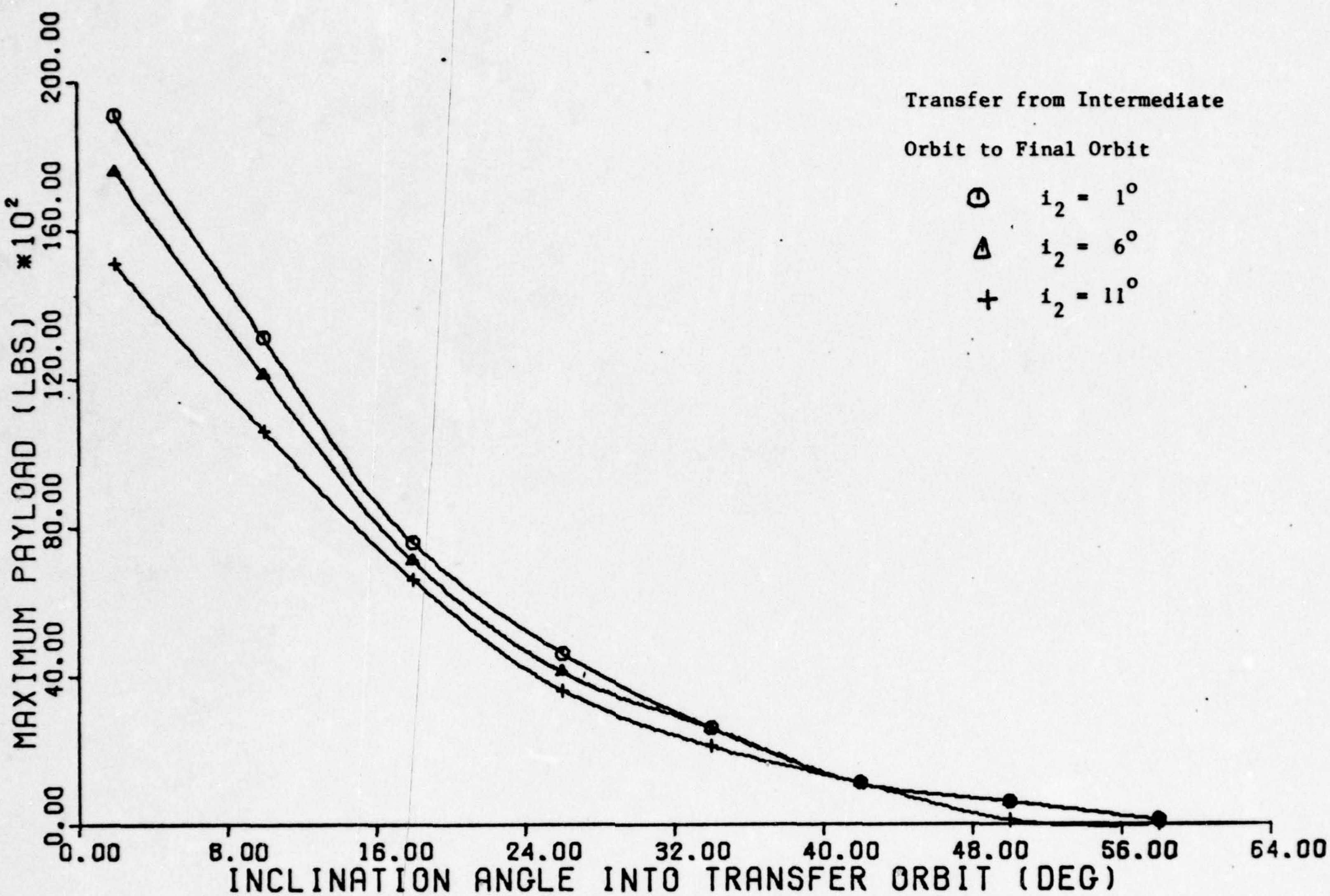


Fig. 9. Maximum Payload vs. Inclination Angle into Transfer Orbit (Payload #1 = 8000 lbs, R_2 = 2.8 DU, R_3 = 5.6 DU, θ = 170^o)

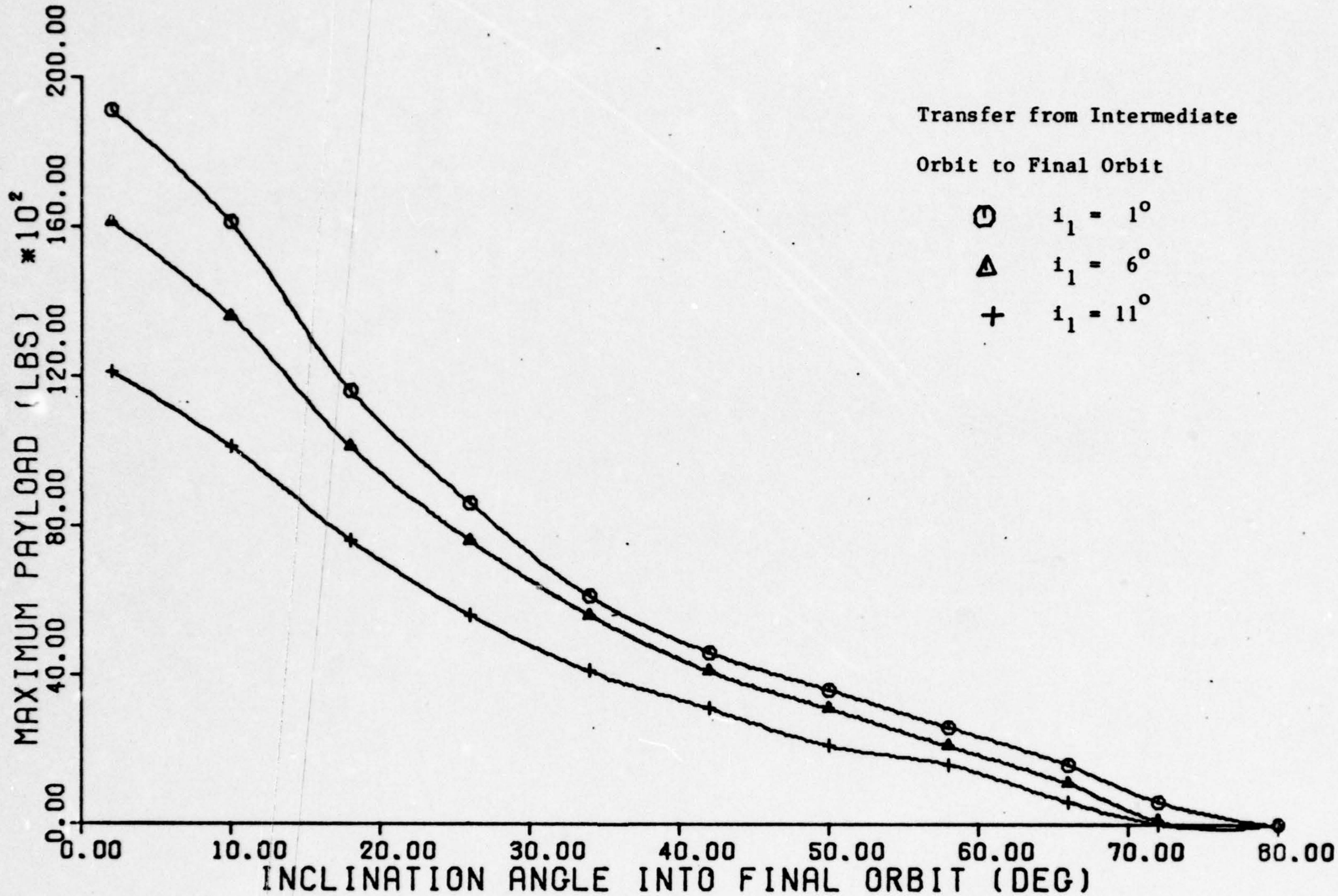


Fig. 10. Maximum Payload vs. Inclination Angle into Final Orbit
(Payload #1 = 4000 lbs, R_2 = 2.8 DU, R_3 = 5.6 DU, θ = 170°)

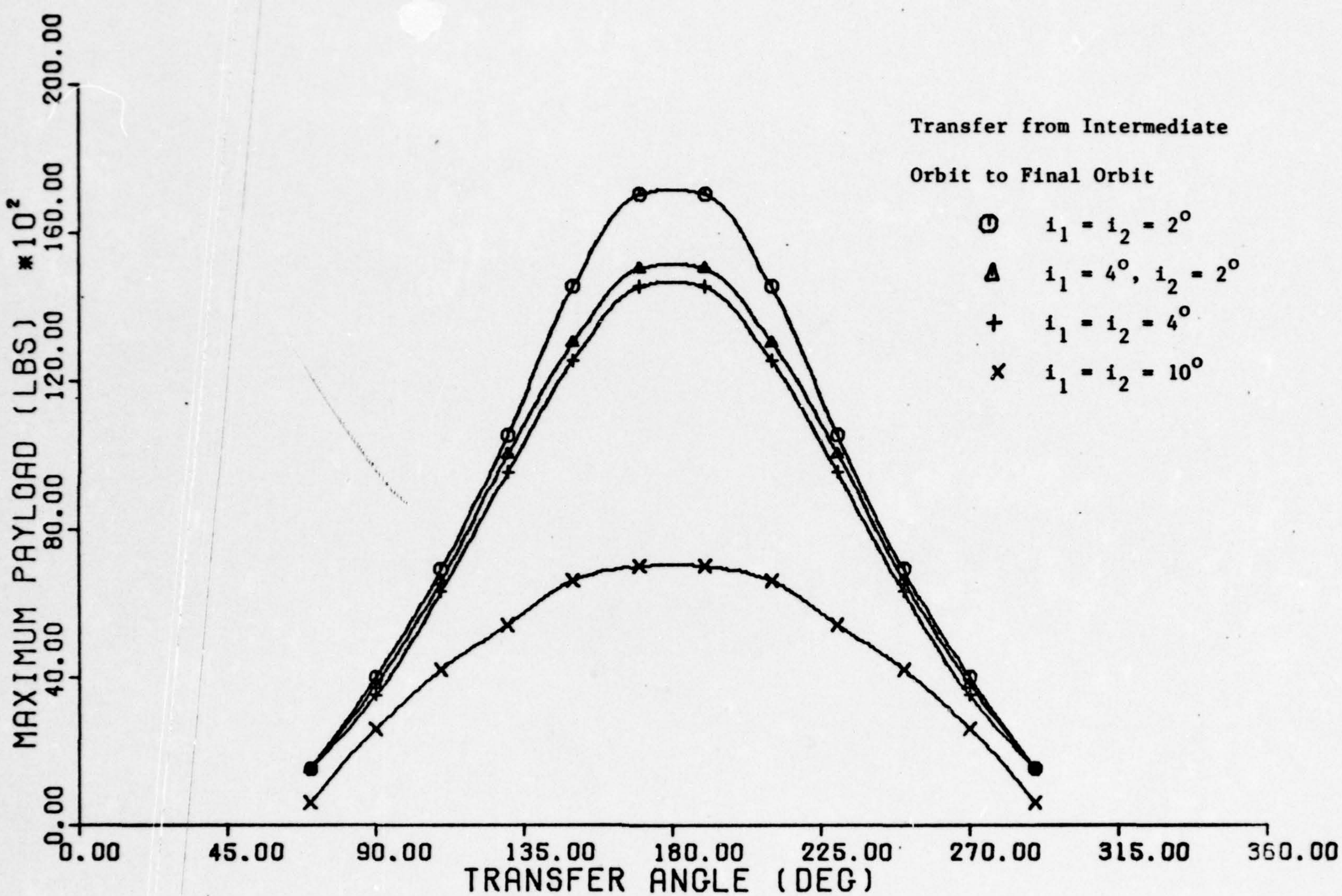


Fig. 11. Maximum Payload vs. Transfer Angle
(Paylaod #1 = 1000 lbs, R_2 = 1.5 DU, R_3 = 2.5 DU)

Table IV. Maximum Payload vs. Transfer Angle, Transfer from Intermediate Orbit to Final Orbit (Payload #1 = 1000 lbs, R_2 = 1.5 DU, $i_1 = i_2 = 2^o$)

Transfer Angle, θ (Deg)	Maximum Payload (LBS)		
	R_3 = 2.5 DU	R_3 = 3.5 DU	R_3 = 5.6 DU
70	1,500	No Transfer	No Transfer
80	2,500	No Transfer	No Transfer
90	4,000	1,000	No Transfer
100	5,500	1,800	0
110	6,900	2,600	1,000
120	8,900	3,800	1,400
130	10,500	5,000	2,200
140	12,500	6,200	3,000
150	14,500	7,400	3,800
160	16,000	8,200	4,600
170	17,000	9,000	5,000

Discontinuity in coefficients exists at $\theta = 180^o$. Symmetry of payload exists for $\theta > 180^o$.

V. Conclusions and Recommendations

Summary of Conclusions

Utilizing an expendable, four-stage vehicle, this thesis has explored the maximum payload capabilities of one possible configuration of a Burner II type vehicle to perform non-coplanar orbit-to-orbit transfers. The maximum payload values for transport from a low earth orbit to orbits at approximately geosynchronous altitude are included. This range includes a majority of missions planned for utilization of the Space Transportation System during the decade of the 1980s.

An impulsive thrust assumption allowed calculation of maximum payload weights to be obtained for a range of geometrical parameters within the performance capabilities of a four-stage solid propellant propulsive upper stage. This impulsive thrust assumption allowed the four-impulse, maximum payload problem to be considered as two dual-impulse problems, whose solutions were in the form of two quartic equations in the square root of the semi-latus rectum of the transfer orbit. Geometry considerations and constraints on the available impulse permitted investigation of maximum payload transfer trajectories in light of the following parameters:

1. The angle between the planes in which the terminal orbits lie,
2. The radii of the terminal orbits,
3. The inclination angle from the initial orbit into the transfer orbit,
4. The inclination angle from the transfer orbit into

the target orbit, and

5. The transfer angle between the radius vector to the point of departure in the initial orbit and the radius vector to the point of arrival in the target orbit.

Results of the parameter study reveal an additional mission flexibility beyond that of a Hohmann-type transfer for a solid rocket motor IUS vehicle. Many non-Hohmann orbit transfers are within the capabilities of vehicles like the Boeing Burner II. They offer considerable savings in mission time. Through some of these non-Hohmann transfers sufficient payload mass can be delivered to make this vehicle a worthy choice for earth orbital missions using the Space Transportation System.

Recommendations for Further Study

The results obtained in this study can be extended in the following manner:

1. Use the more realistic theory for finite thrust to set up a nonlinear two-point boundary value problem. By numerical methods, the solution will result in more realistic values of the maximum transferrable payload.

2. Investigate both impulsive and finite thrust solutions for orbits that are not circular but exhibit some eccentricity.

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Appendix A

Expressions for the Calculation of Impulses

If the total vehicle system mass, M, is defined as

M = M1 + M2 + PAYLOAD #1 + M3 + M4 + PAYLOAD #2

where

M1 = (M1)propellant + (M1)inert

M2 = (M2)propellant + (M2)inert

M3 = (M3)propellant + (M3)inert

M4 = (M4)propellant + (M4)inert

then, the impulses for each stage are given by the following:

$$\Delta V_1 = \frac{(\text{Thrust})_1(\text{Burn Time})_1}{(\text{M1})\text{prop}} \ln\left[\frac{\text{M}}{\text{M} - (\text{M1})\text{prop}}\right]$$

$$\Delta V_2 = \frac{(\text{Thrust})_2(\text{Burn Time})_2}{(\text{M2})\text{prop}} \ln\left[\frac{\text{M} - \text{M1}}{\text{M} - \text{M1} - (\text{M2})\text{prop}}\right]$$

$$\Delta V_3 = \frac{(\text{Thrust})_3(\text{Burn Time})_3}{(\text{M3})\text{prop}} \ln\left[\frac{\text{M} - \text{M1} - \text{M2} - \text{Pay\#1}}{\text{M} - \text{M1} - \text{M2} - \text{Pay\#1} - (\text{M3})\text{prop}}\right]$$

$$\Delta V_4 = \frac{(\text{Thrust})_4(\text{Burn Time})_4}{(\text{M4})\text{prop}} \ln\left[\frac{\text{M} - \text{M1} - \text{M2} - \text{Pay\#1} - \text{M3}}{\text{M} - \text{M1} - \text{M2} - \text{Pay\#1} - \text{M3} - (\text{M4})\text{prop}}\right]$$

Appendix B

Expressions for Coefficients of the Quartic Equation

$$Ay^4 + By^3 + Cy^2 + Dy + E = 0$$

where $y = \ell_T^{1/2}$.

$$A = \beta_1^2 - 2I_T^2\beta_4$$

$$B = 2\beta_1\beta_3 - 2I_T^2\beta_5$$

$$C = 2\beta_1\beta_2 + \beta_3^2 - 2I_T^2\beta_8 + I_T^4$$

$$D = 2\beta_2\beta_3 - 2I_T^2\beta_6$$

$$E = \beta_2^2 - 2I_T^2\beta_7$$

$$\beta_1 = \alpha_2 - \alpha_6$$

$$\beta_2 = \alpha_3 - \alpha_7$$

$$\beta_3 = \alpha_5 - \alpha_8$$

$$\beta_4 = 2\alpha_1$$

$$\beta_5 = \alpha_2 + \alpha_6$$

$$\beta_6 = \alpha_3 + \alpha_7$$

$$\beta_7 = 2\alpha_4$$

$$\beta_8 = \alpha_5 + \alpha_8$$

$$\alpha_1 = \omega_2^2 + \omega_3^2$$

$$\alpha_2 = -2(\omega_2 V_{x_1} + \omega_3 V_{y_1} \cos i_1)$$

$$\alpha_3 = -2\omega_1 V_{x_1}$$

$$\alpha_4 = \omega_1^2$$

$$\alpha_5 = V_1^2 + 2\omega_1\omega_2$$

$$\alpha_6 = -2(\omega_4 V_{x_2} + \omega_5 V_{y_2} \cos i_2)$$

$$\alpha_7 = 2\omega_1 V_{x_2}$$

$$\alpha_8 = V_2^2 - 2\omega_1\omega_4$$

$$\omega_1 = \sqrt{\mu}\,\frac{(1 - \cos\theta)}{\sin\theta}$$

$$\omega_2 = \frac{\sqrt{\mu}\,(R_2\cos\theta - R_1)}{R_1R_2\sin\theta}$$

$$\omega_3 = \frac{\sqrt{\mu}}{R_1}$$

$$\omega_4 = \frac{\sqrt{\mu}\,(R_2 - R_1\cos\theta)}{R_1R_2\sin\theta}$$

$$\omega_5 = \frac{\sqrt{\mu}}{R_2}$$

Appendix C

Plots of Computer Results for a Second Vehicle

Table V. Another Model Vehicle's Weights and Propulsion Summary

Total Orbiter Payload Capacity	65,000 LB
Total IUS Weight	51,000 LB
Stage 1	
Stage Weight	24,000 LB
Propellant Weight	20,000 LB
Total Inert Weight	4,000 LB
Average Thrust	42,000 LB
Average Burn Time	140 SEC
Stage 2	
Stage Weight	12,000 LB
Propellant Weight	9,500 LB
Total Inert Weight	2,500 LB
Average Thrust	14,000 LB
Average Burn Time	100 SEC
Stage 3	
Stage Weight	12,000 LB
Propellant Weight	9,500 LB
Total Inert Weight	2,500 LB
Average Thrust	14,000 LB
Average Burn Time	100 SEC
Stage 4	
Stage Weight	3,000 LB
Propellant Weight	2,300 LB
Total Inert Weight	700 LB
Average Thrust	6,000 LB
Average Burn Time	60 SEC

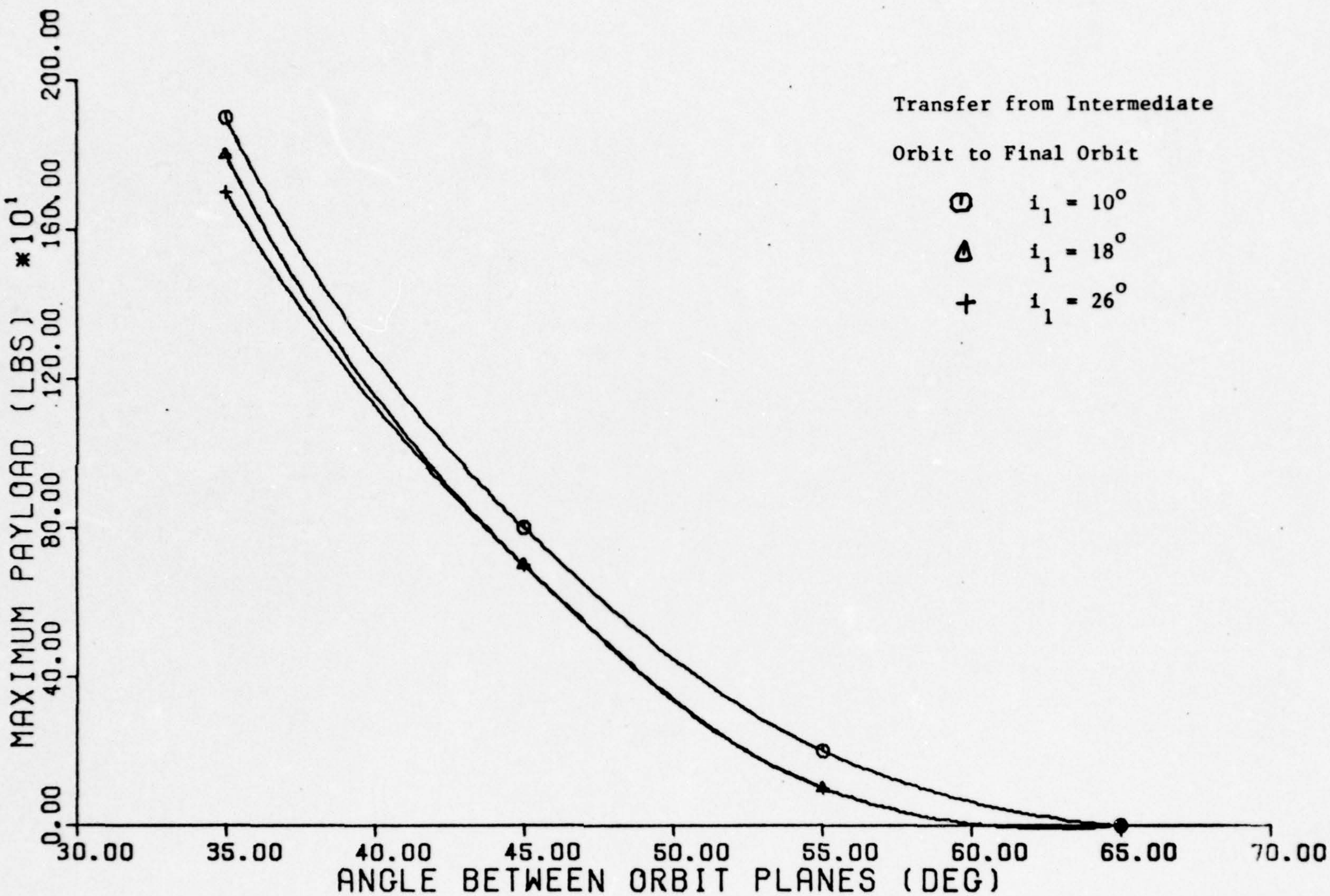

Fig. 12. Maximum Payload vs. Angle Between Orbit Planes
(Payload #1 = 1000 lbs, $R_2 = R_3 = 4.0$ DU, $\phi = 170^o$)

Table VI. Maximum Payload vs. Angle Between Orbit Planes
(Payload #1 = 1000 lbs, $R_2 = R_3 = 4.0$ DU, $\phi = 170^o$)

Transfer from Intermediate Orbit to Final Orbit				
Angle Between Orbit Planes (Deg)	$i_1 = 10^o$			
	i_2 (Deg)	θ (Deg)	ψ (Deg)	Payload (LBS)
25	15.2	163.8	6.6	4300
35	25.2	166.5	4.1	1900
45	35.2	167.7	3.0	800
55	45.2	168.4	2.4	200
65	55.2	169.0	2.1	0

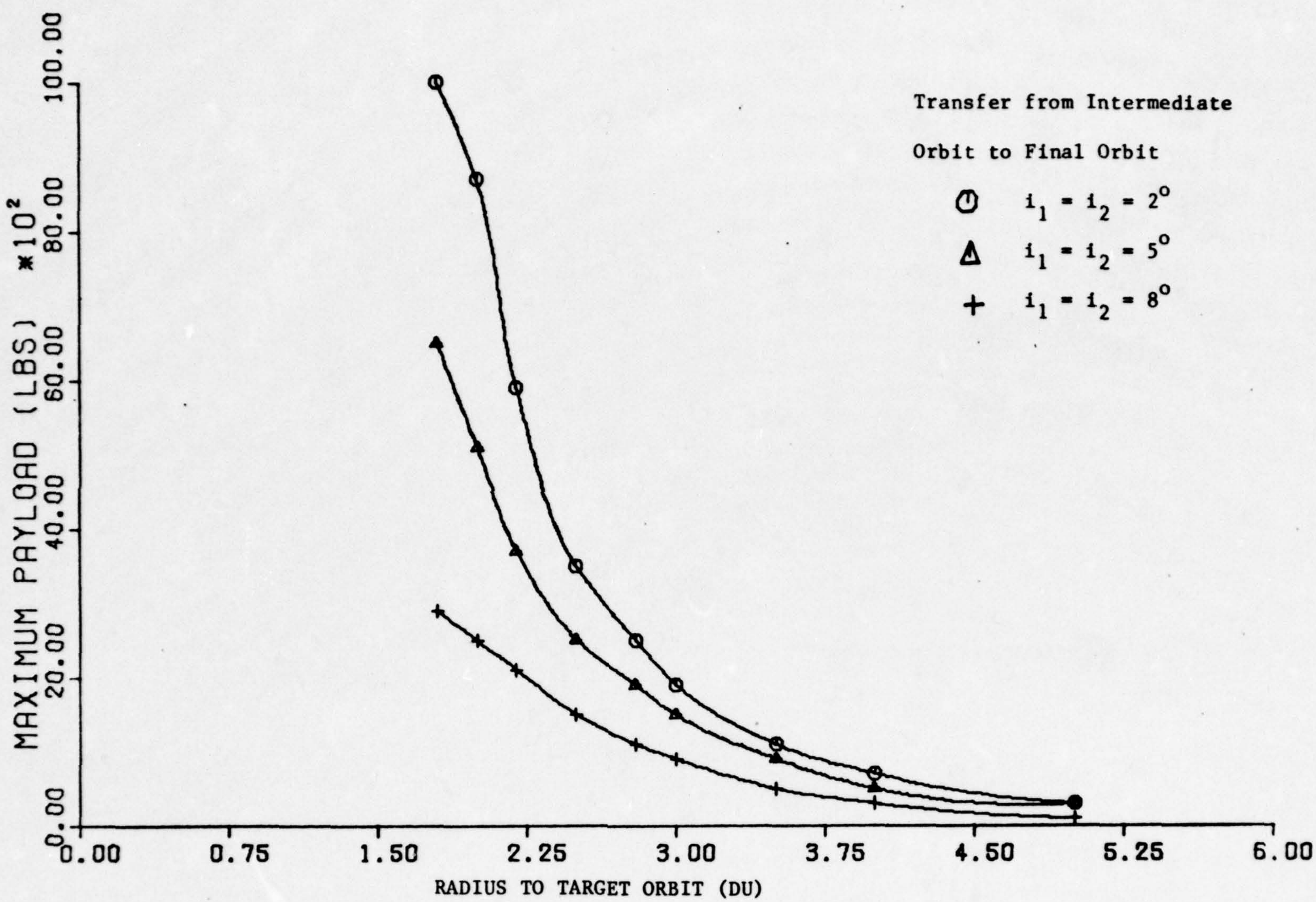

Fig. 13. Maximum Payload vs. Radius to Target Orbit
(Payload #1 = 1000 lbs, R_2 = 1.5 DU, $\theta = 150^o$)

Table VII. Maximum Payload vs. Radius to Target Orbit Transfer from Intermediate Orbit to Final Orbit (Payload #1 = 1000 lbs, R_2 = 1.5 DU)

Radius to Target Orbit, R_3 (DU)	Maximum Payload (LBS) for $i_1 = i_2 = 5^o$	
	$\theta = 150^o$	$\theta = 165^o$
1.8	6,500	6,900
2.0	5,100	5,500
2.2	3,700	4,300
2.5	2,500	3,100
2.8	1,900	2,300
3.0	1,500	1,900
3.5	900	1,300
4.0	500	900
5.0	300	500

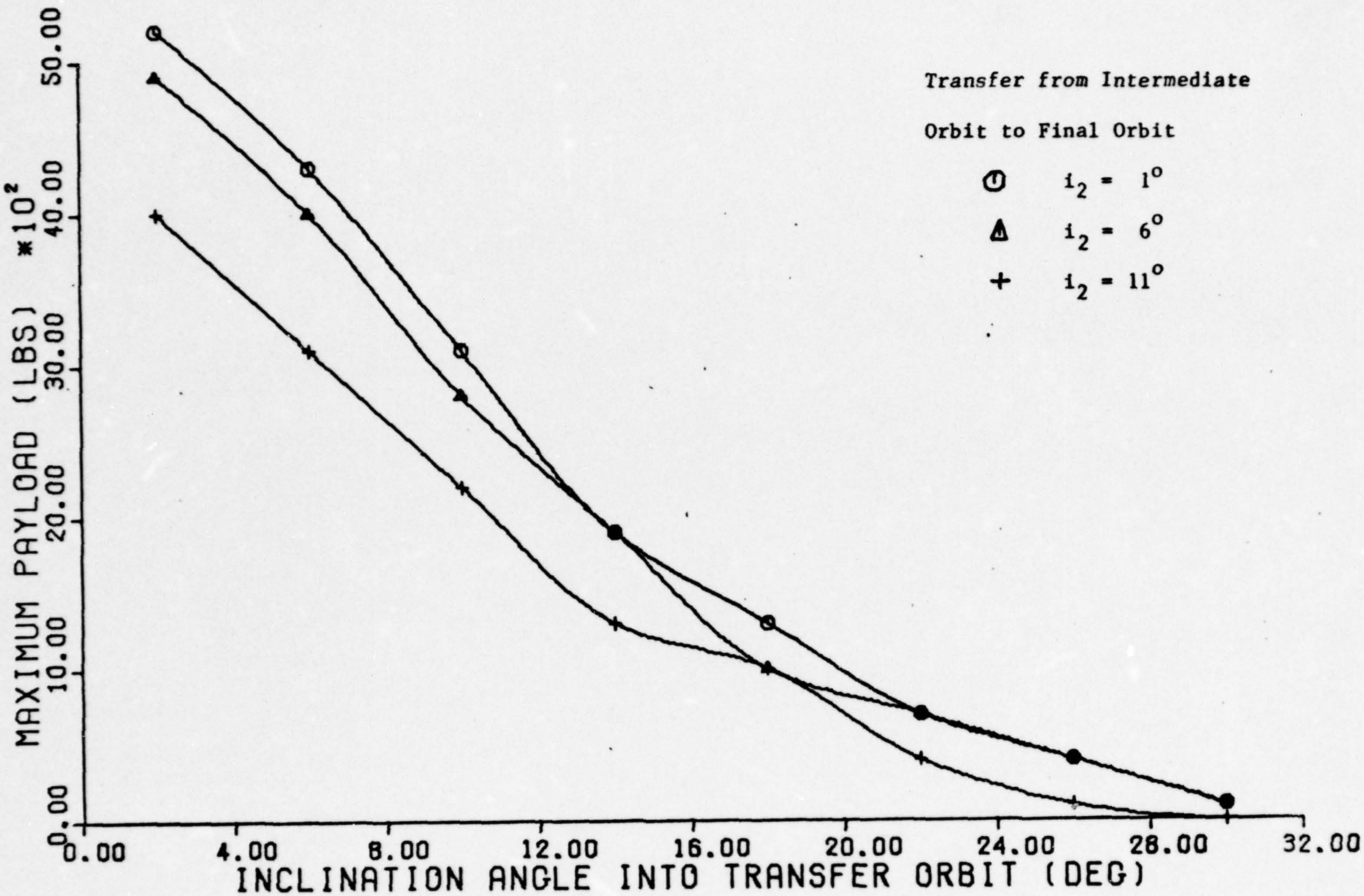

Fig. 14. Maximum Payload vs. Inclination Angle into Transfer Orbit
(Payload #1 = 8000 lbs, R_2 = 2.8 DU, R_3 = 5.6 DU, θ = 170°)

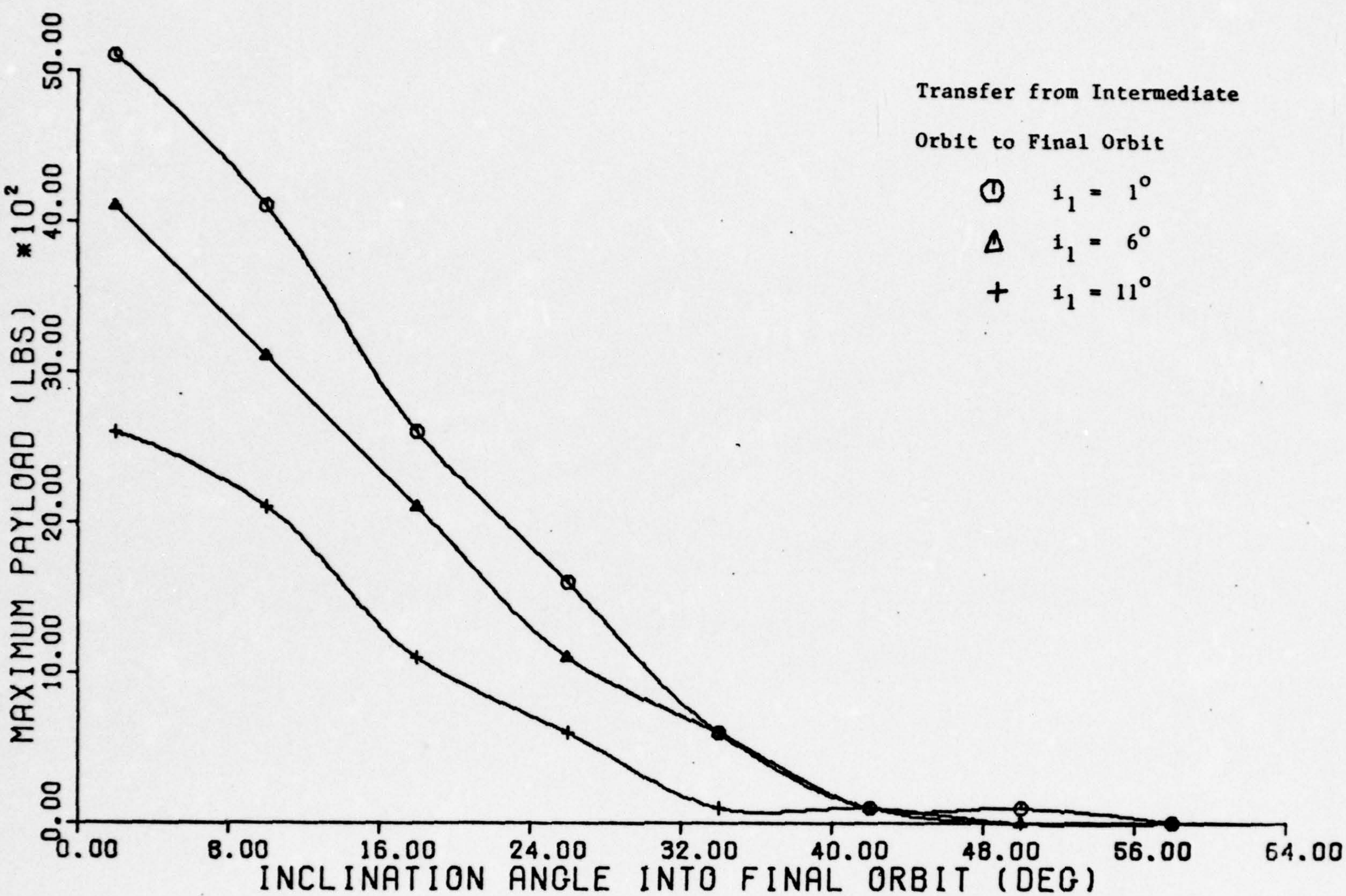

Fig. 15. Maximum Payload vs. Inclination Angle into Final Orbit
(Payload #1 = 4000 lbs, R_2 = 2.8 DU, R_3 = 5.6 DU, θ = 170°)

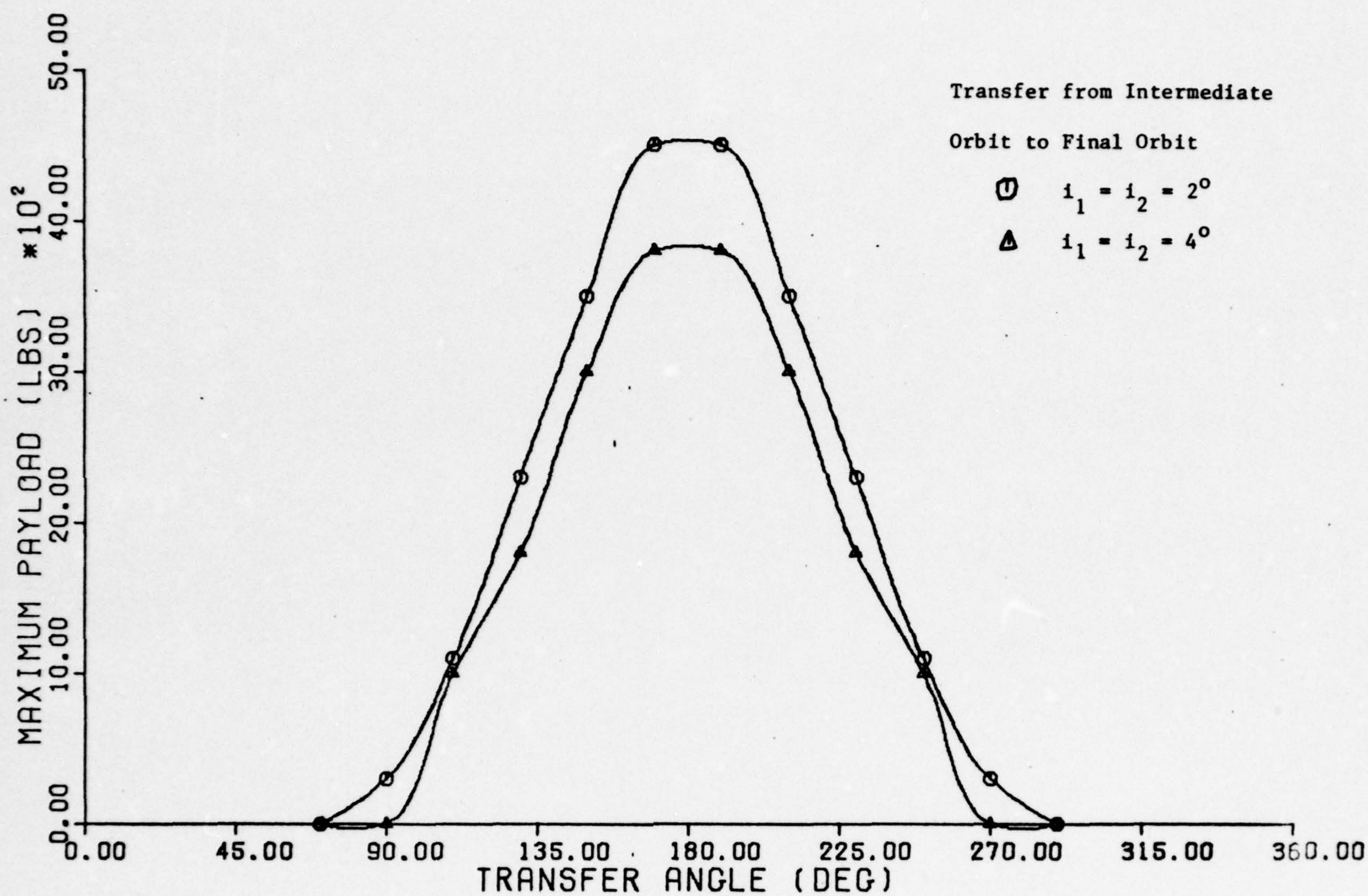

Fig. 16. Maximum Payload vs. Transfer Angle
(Payload #1 = 1000 lbs, R_2 = 1.5 DU, R_3 = 2.5 DU)

Table VIII. Maximum Payload vs. Transfer Angle
Transfer from Intermediate Orbit to Final Orbit
(Payload #1 = 1000 lbs, R_2 = 1.5 DU, $i_1 = i_2 = 2^o$)

Transfer Angle, θ (DU)	Maximum Payload (LBS)	
	R_3 = 2.5 DU	R_3 = 3.5 DU
80	0	No Transfer
90	300	No Transfer
100	700	No Transfer
110	1,100	No Transfer
120	1,700	0
130	2,300	0
140	2,900	0
150	3,500	1,000
160	4,100	1,400
170	4,500	1,800
Discontinuity in coefficients exists at $\theta = 180^o$. Symmetry of payload values exist for $\theta > 180^o$.		

Appendix D

Listing of Computer Program

In the pages of this appendix appear the computer program that was used to obtain data for this thesis. In order that reading these pages might be alittle easier the following synonyms are defined:

M1	stage weight of SRM #1 (lbs)
M2	stage weight of SRM #2 (lbs)
M3	stage weight of SRM #3 (lbs)
M4	stage weight of SRM #4 (lbs)
M	total upper stage vehicle weight
LOAD	1 means payload to be unloaded into intermediate orbit
	2 means payload to be unloaded into final orbit
PAY	designated payload mass
TH1	average thrust of SRM #1 (lbs)
TH2	average thrust of SRM #2 (lbs)
TH3	average thrust of SRM #3 (lbs)
TH4	average thrust of SRM #4 (lbs)
BT1	average burn time of SRM #1 (sec)
BT2	average burn time of SRM #2 (sec)
BT3	average burn time of SRM #3 (sec)
BT4	average burn time of SRM #4 (sec)
PDAES1	parking orbit distance above earth's surface (DU)
PDAES2	intermediate orbit distance above earth's surface (DU)
PDAES3	final orbit distance above earth's surface (DU)
EX1	eccentricity of parking orbit
EX2	eccentricity of intermediate orbit
EX3	eccentricity of final orbit
EXT	eccentricity of transfer orbit
XINC1	inclination angle between initial orbit and transfer arc
XINC2	inclination angle between transfer arc and target orbit
THETA	angle between position vectors
SMAXIS	semi-major axis
TIMP	total impulse used
XIMP/SIMP	total impulse available
M1P	propellant weight of SRM #1 (lbs)
M1I	inert weight of SRM #1 (lbs)
M2P	propellant weight of SRM #2 (lbs)
M2I	inert weight of SRM #2 (lbs)
M3P	propellant weight of SRM #3 (lbs)
M3I	inert weight of SRM #3 (lbs)
M4P	propellant weight of SRM #4 (lbs)
M4I	inert weight of SRM #4 (lbs)

```
      PROGRAM MAIN (INPUT,OUTPUT,TAPE6=OUTPUT,PLOT)
      COMMON/POLY/W1,W2,W3,W4,W5,ALPHA1,ALPHA2,ALPHA3,ALPHA4,ALPHA5,
     1ALPHA6,ALPHA7,ALPHA8
      COMMON/COF/ BETA1,BETA2,BETA3,BETA4,BETA5,BETA6,BETA7,BETA8
      COMMON/TRAC/VTX1,VTY1,VTX2,VTY2,VTX3,VTY3,EXT,SMAXIS,ZZ
      DIMENSION XINC1(370),THETA(370),THETB(370),SQRP(4),SRRP(4)
      DIMENSION              XINC2(370), ZZ(4),          PAY1(370),
     1PAY2(370), MW1(370)
      DOUBLE PRECISION COE(5),ROOTR(4), ROOTI(4)
      INTEGER ORBIT
      REAL NU1,M,NU2,M1,M2,M3,M4,MW1,MS
C            DIMENSIONS TO INPUT ARE POUNDS,DEGREES,AND DU
      READ*,EX1,     PDAES1,A,EX2,PDAES2,       NU1,DELTA
      READ*,TH1,TH2,TH3,TH4,BT1,BT2,BT3,BT4
      READ*,   M1,M2,M3,M4,LOAD,PAY
      SLUG = 32.174
      TU = 1.239444E-3
      DU = 2.903656E-4
      RAD = 57.2957795
      DUTJ = 2.593627354E4
      PRINT*
      PRINT*
      WRITE(5,89)
89    FORMAT(35X,*------ FIRST ORBITAL TRANSFER ------*)
      WRITE(6,88) LOAD,PAY
88    FORMAT(36X,*THE*,1X,I1,1X,*PAYLOAD = *,E15.8,1X,*LBS*)
      PRINT*
      PRINT*
      PDAES1 = PDAES1 / DU
      PDAES2 = PDAES2 / DU
      WRITE(5,90) EX1,     PDAES1,A,EX2,PDAES2
90    FORMAT( 5X,*ECCENTRICITY OF PARKING ORBIT = *,E15.8/,
     25X,*PERIGEE DISTANCE ABOVE EARTH SURFACE OF PARKING ORBIT = *,
     2E15.8,2X,*N.MI.*,/,
     35X,*ANGLE BETWEEN ORBITAL PLANES = *,E15.8,2X,*DEG*,/,
```

```
     45X,*ECCENTRICITY OF TARGET ORBIT = *,E15.8/,
     55X,*PERIGEE DISTANCE ABOVE EARTH SURFACE OF TARGET ORBIT = *,
     5E15.8,2X,*N.MI.*)
      WRITE(6,204) M1,M2,M3,M4
204   FORMAT(5X,*1-2-3-4 STAGE WEIGHTS(LBS):*,4(E15.8,2X))
      WRITE(6,205) TH1,TH2,TH3,TH4,BT1,BT2,BT3,BT4
205   FORMAT(5X,*AVG THRUST AT TERMINAL POINTS 1-4 (LBS):*,2X,4E12.5,/,
     15X,*AVG BURN TIMES AT TERMINAL POINTS 1-4 (SEC):*,2X,4E12.5)
      PRINT*
      PRINT*
      PDAES1 = PDAES1 * DU
      PDAES2 = PDAES2 * DU
      REARTH = 1.0
      A = A/RAD
C              THE INITIAL POSITION IS DEFINED AND INITIAL VELOCITIES
C              CALCULATED
      RP1 = REARTH + PDAES1
      P1 = RP1 * (1. + EX1)
      M1 = M1 / SLUG
      M2 = M2 / SLUG
      M3 = M3 / SLUG
      M4 = M4 / SLUG
      PAY = PAY / SLUG
      M = M1 + M2 + M3 + M4 + PAY
      XM = M
      SM = M
      DO 7 IJ = 179,180,2
      IF(THETA(IJ).EQ.180) GO TO 7
      R1 = P1/(1. + EX1 * COS(NU1))
      DELTA = DELTA/RAD
      VY1 = SQRT(P1)/R1
      VSOL = ((EX1 ** 2 - 1.)/P1) + 2./R1
      VX1 = 0.0
C              DETERMINE IF RADIAL VELOCITY IS OUTWARD OR INWARD
      PI = 3.141592654
      IF(NU1.LT.0.0) VX1 = -VX1
      IF(NU1.GT.PI.AND.NU1.LT.2.*PI) VX1 = -VX1
```

```
C              THE TARGET POSITION IS DEFINED AND FINAL RADIUS AND
C              VELOCITIES CALCULATED
      DO 8 J = 1,11,1
      XINC1(J) = 0.0174532925 * J
      DO 13 KI = 1,2,1
      XINC2(KI) = 1. * KI
      THETA(IJ) = 1. * IJ
      RP2 = REARTH + PDAES2
      P2 = RP2 * (1. + EX2)
      R2 = P2/(1. + EX2 * COS(DELTA - THETA(IJ)))
      R2 = R2/DU
      XINC1(J) = XINC1(J) * RAD
      R1 = R1/DU
      WRITE(5,83) R1
83    FORMAT( 5X,*THE DEPARTURE POINT IS AT*,2X,E15.8,2X,*N.MI.*)
      WRITE(5,84) R2,XINC1(J)
84    FORMAT( 5X,*THE ARRIVAL POINT IS AT*,2X,E15.8,2X,*N.MI.*,/,
     15X,*THE INCLINATION ANGLE FROM PARKING ORBIT TO TRANSFER ARC IS*,
     22X,E15.8,2X,*DEG*)
      XINC1(J) = XINC1(J)/RAD
      R2 = R2 * DU
      R1 = R1 * DU
      VY2 = SQRT(P2)/R2
      VSQ2 = ((EX2 ** 2 - 1.)/P2) + 2./R2
      VX2 = 0.0
      VY1 = VY1 * DUTU
      VX1 = VX1 * DUTU
      VY2 = VY2 * DUTU
      VX2 = VX2 * DUTU
      WRITE(5,200) VX1,VY1
200   FORMAT( 5X,*X-COMPONENT OF VELOCITY IN INITIAL ORBIT = *,E15.8,
     12X,*FT/SEC*,/,5X,*Y-COMPONENT OF VELOCITY IN INITIAL ORBIT = *,
     2E15.8,2X,*FT/SEC*)
      WRITE(5,201) XINC2(KI), THETA(IJ)
201   FORMAT( 5X,*INCLINATION ANGLE FROM TRANSFER TO TGT ORBIT = *,
     1E15.8,2X,*DEG*,/,5X,*ANGLE BETWEEN R1 AND R2 = *,E15.8,2X,*DEG*)
```

```
      WRITE(6,202) VX2,VY2
202   FORMAT(5X,*X-COMPONENT OF VELOCITY IN TGT ORBIT = *,E15.8,2X,
     1*FT/SEC*,/,5X,*Y-COMPONENT OF VELOCITY IN TGT ORBIT = *,E15.8,2X,
     2*FT/SEC*)
      VY1 = VY1 / DUTU
      VX1 = VX1 / DUTU
      VY2 = VY2 / DUTU
      VX2 = VX2 / DUTU
      XINC2(KI) = XINC2(KI) / RAD
      THETA(IJ) = THETA(IJ) / RAD
      WRITE(6,100)
100   FORMAT(30X,*COMPLEX ROOTS TO INTERMEDIATE ORBIT*)
      PRINT*
      WRITE(6,101)
101   FORMAT( 9X,*IMPULSE*,11X,*FIRST*,10X,*SECOND*,10X,*THIRD*,5X,
     110X,*FOURTH*,10X,*PAYLOAD*)
      WRITE(6,104)
104   FORMAT(9X,*(DU/TU)*,12X,*(DU)*,11X,*(DU)*,12X,*(DU)*,11X,*(DU)*,
     112X,*(LBS)*)
      PRINT*
      M = XM
C             VARY THE TOTAL WEIGHT OF THE IUS VEHICLE
      DO 10 K = 1,301,10
      XIMP = (((TH1 * BT1)/M1P)*(LN(M/(M-M1P))) + ((TH2 * BT2)/M2P)*(LN(M-M1)/(M-M1-M2P)))
C             THE COEFFICIENTS OF THE FOURTH ORDER POLYNOMIAL OF SQUARE ROOT OF
C             SEMI-LATUS RECTUM ARE COMPUTED
      CALL COEFF(R1,R2,THETA(IJ),XINC1(J),XINC2(KI),VX1,VY1,VX2,VY2,
     1XIMP,COE)
C             THE POLYNOMIAL IS FACTORED
      N = 4
      CALL DMULR(COE,N,ROOTR,ROOTI)
C             THE ROOTS OBTAINED ARE SEPARATED AND JUST REAL ROOTS ARE USED TO
C             FLY THE POSSIBLE TRAJECTORY BY CALCULATING THE TRANSFER VELOCITY
C             AND OTHER PARAMETERS ON THE TRANSFER ARC
```

```
      KJ = 0
      DO 45 IK = 1,N
      IF(DABS(ROOTR(IK)).EQ.0.0) GO TO 45
      IF(DABS(ROOTI(IK)).GT.1.E-6) GO TO 45
      KJ = KJ + 1
      SQRP(KJ) = SNGL(ROOTR(IK))
      CALL TRAJ(SQRP(KJ), R1,XIMP,KJ)
45    CONTINUE
      IF(ZZ(1).GT.1.E-6.AND.ZZ(2).GT.1.E-6.AND.ZZ(3).GT.1.E-6.AND.
     1ZZ(4).GT.1.E-6) GO TO 10
      IF(DABS(ROOTI(1)).GT.1.E-6.AND.DABS(ROOTI(2)).GT.1.E-6.AND.
     1DABS(ROOTI(3)).GT.1.E-6.AND.DABS(ROOTI(4)).GT.1.E-6) GO TO 10
C           THE PAYLOAD IS DETERMINED
      MW1(J) = M - M1 - M2
      IF(LOAD.EQ.1) GO TO 50
      IF(LOAD.EQ.2) GO TO 51
50    PAY1(J) = PAY
      MW1(J) = MW1(J) * SLUG
      WRITE(6,108) XIMP,(ROOTR(JK),JK=1,4),MW1(J)
108   FORMAT(5X,E15.8,2X,4E15.8,2X,E15.8)
      WRITE(6,109)(ROOTI(JK),JK= 1,4)
109   FORMAT(22X,4E15.8)
      GO TO 52
51    PAY1(J) = MW1(J) - M3 - M4 - PAY
      PAY1(J) = PAY1(J) * SLUG
      WRITE(6,102) XIMP,(ROOTR(JK),JK=1,4),PAY1(J)
102   FORMAT(5X,E15.8,2X,4E15.8,2X,E15.8)
      WRITE(6,103)(ROOTI(JK),JK= 1,4)
103   FORMAT(22X,4E15.8)
      M = M + 3.1081 * K
10    CONTINUE
 13   CONTINUE
8     CONTINUE
 7    CONTINUE
```

```
      PRINT*
      PRINT*
      WRITE(6,106)
106   FORMAT(35X,*------ SECOND ORBITAL TRANSFER ------*)
      PAY = PAY * SLUG
      WRITE(6,105) LOAD,PAY
105   FORMAT(36X,*THE*,1X,I1,1X,*PAYLOAD = *,E15.8,1X,*LBS*)
      PRINT*
      PRINT*
      READ*,B,EX3,PDAES3,NU2,DELTB
      PDAES2 = PDAES2/DU
      PDAES3 = PDAES3/DU
      WRITE(6,390) EX2,     PDAES2, B,EX3,PDAES3
390   FORMAT( 5X,*ECCENTRICITY OF INTERIM ORBIT = *,E15.8/,
     25X,*PERIGEE DISTANCE ABOVE EARTH SURFACE OF INTERIM ORBIT = *,
     2E15.8,2X,*N.MI.*,/,
     35X,*ANGLE BETWEEN ORBITAL PLANES = *,E15.8,2X,*DEG*,/,
     45X,*ECCENTRICITY OF TARGET ORBIT = *,E15.8/,
     55X,*PERIGEE DISTANCE ABOVE EARTH SURFACE OF TARGET ORBIT = *,
     5E15.8,2X,*N.MI.*)
      M1 = M1 * SLUG
      M2 = M2 * SLUG
      M3 = M3 * SLUG
      M4 = M4 * SLUG
      WRITE(6,403) M1,M2,M3,M4
403   FORMAT(5X,*1-2-3-4 STAGE WEIGHTS(LBS):*,4(E15.8,2X))
      PRINT*
      PRINT*
      PAY = PAY / SLUG
      M1 = M1 / SLUG
      M2 = M2 / SLUG
      M3 = M3 / SLUG
      M4 = M4 / SLUG
      M = M/ SLUG
      PDAES2 = PDAES2 * DU
      PDAES3 = PDAES3 * DU
```

```
      RP3 = REARTH + PDAES3
      P3 = RP3 * (1. + EX3)
      R3 = P3/(1. + EX3 * COS(DELTB - THETB(IJ)))
      R3 = R3/DU
      XINC1(J) = XINC1(J) * RAD
      R2 = R2/DU
      WRITE(6,183) R2
183   FORMAT( 5X,*THE DEPARTURE POINT IS AT*,2X,E15.8,2X,*N.MI.*)
      WRITE(6,184) R3,XINC1(J)
184   FORMAT( 5X,*THE ARRIVAL POINT IS AT*,2X,E15.8,2X,*N.MI.*,/,
     15X,*THE INCLINATION ANGLE FROM INTERIM ORBIT TO TRANSFER ARC IS*,
     22X,E15.8,2X,*DEG*)
      XINC1(J) = XINC1(J)/RAD
      R2 = R2 * DU
      R3 = R3 * DU
      VY3 = SQRT(P3)/R3
      VSQ3 = ((EX3 ** 2 - 1.)/P3) + 2./R3
      VX3 = 0.0
      VX2 = VX2 * DUTU
      VY2 = VY2 * DUTU
      VX3 = VX3 * DUTU
      VY3 = VY3 * DUTU
      B = B/RAD
      DELTB = DELTB/RAD
      DO 17 IJ = 179,180,2
      IF(THETB(IJ).EQ.180) GO TO 17
      R2 = P2/(1. + EX2 * COS(NU2))
C           DETERMINE IF RADIAL VELOCITY IS OUTWARD OR INWARD
      IF(NU2.LT.0.0) VX2 = - VX2
      IF(NU2.GT.PI.AND.NU2.LT.2.*PI) VX2 = -VX2
C           THE TARGET POSITION IS DEFINED AND FINAL RADIUS AND
C           VELOCITIES CALCULATED
      DO 3 J = 1,11,1
      XINC1(J) = 0.0174532925 * J
      DO 23 KI = 1,2,1
      XINC2(KI) = 1. * KI
      THETB(IJ) = 1. * IJ
```

```
      WRITE(6,400) VX2,VY2
400   FORMAT( 5X,*X-COMPONENT OF VELOCITY IN INITIAL ORBIT = *,E15.8,
     12X,*FT/SEC*,/,5X,*Y-COMPONENT OF VELOCITY IN INITIAL ORBIT = *,
     2E15.8,2X,*FT/SEC*)
      WRITE(6,401) XINC2(KI), THETB(IJ)
401   FORMAT( 5X,*INCLINATION ANGLE FROM TRANSFER TO TGT ORBIT = *,
     1E15.8,2X,*DEG*,/,5X,*ANGLE BETWEEN R2 AND R3 = *,E15.8,2X,*DEG*)
      WRITE(6,402) VX3,VY3
402   FORMAT(5X,*X-COMPONENT OF VELOCITY IN TGT ORBIT = *,E15.8,2X,
     1*FT/SEC*,/,5X,*Y-COMPONENT OF VELOCITY IN TGT ORBIT = *,E15.8,2X,
     2*FT/SEC*)
      M = SM
      VX2 = VX2 / DUTU
      VY2 = VY2 / DUTU
      VX3 = VX3 / DUTU
      VY3 = VY3 / DUTU
      XINC2(KI) = XINC2(KI) / RAD
      THETB(IJ) = THETB(IJ) / RAD
      WRITE(6,300)
300   FORMAT(30X,*COMPLEX ROOTS TO FINAL ORBIT*)
      PRINT*
      WRITE(6,301)
301   FORMAT( 9X,*IMPULSE*,11X,*FIRST*,10X,*SECOND*,10X,*THIRD*,
     110X,*FOURTH*,10X,*PAYLOAD*)
      WRITE(6,304)
304   FORMAT(9X,*(DU/TU)*,12X,*(DU)*,11X,*(DU)*,12X,*(DU)*,11X,*(DU)*,
     112X,*(LBS)*)
      PRINT*
      IF(LOAD.EQ.1) GO TO 57
      IF(LOAD.EQ.2) GO TO 58
57    MS = M3 + M4
      GO TO 59
58    MS = M3+M4+PAY
59    CONTINUE
C           VARY THE TOTAL WEIGHT OF THE IUS VEHICLE
```

```
      DO 11 I = 1,321,10
      SIMP = (((TH3 * BT3)/M3P)*(LN(M-M1-M2-PAY1)/M-M1-M2-PAY1-M3P) + ((TH4*BT4)/M4P)*(LN(M4+PAY2)/(M4I))
C           THE COEFFICIENTS OF THE 4TH ORDER POLYNOMIAL OF SQUARE ROOT OF
C           SEMI-LATUS RECTUM ARE COMPUTED.
      CALL COEFF(R2,R3,THETB(IJ),XINC1(J),XINC2(KI),VX2,VY2,VX3,VY3,
     1SIMP,COE)
C           THE POLYNOMIAL IS FACTORED
      N = 4
      CALL DMULR(COE,N,ROOTR,ROOTI)
C           THE ROOTS OBTAINED ARE SEPARATED AND JUST REAL ROOTS ARE USED TO
C           FLY THE POSSIBLE TRAJECTORY BY CALCULATING THE TRANSFER VELOCITY
C           AND OTHER PARAMETERS ON THE TRANSFER ARC
      KJ = 0
      DO 46 IK = 1,N
      IF(DABS(ROOTR(IK)).EQ.0.0) GO TO 46
      IF(DABS(ROOTI(IK)).GT.1.E-6) GO TO 46
      KJ = KJ + 1
      SRRP(KJ) = SNGL(ROOTR(IK))
      CALL TRAJ(SRRP(KJ), R2,SIMP,KJ)
46    CONTINUE
      IF(ZZ(1).GT.1.E-6.AND.ZZ(2).GT.1.E-6.AND.ZZ(3).GT.1.E-6.AND.
     1ZZ(4).GT.1.E-6) GO TO 11
      IF(DABS(ROOTI(1)).GT.1.E-6.AND.DABS(ROOTI(2)).GT.1.E-6.AND.
     1DABS(ROOTI(3)).GT.1.E-6.AND.DABS(ROOTI(4)).GT.1.E-6) GO TO 11
C           THE PAYLOAD IS DETERMINED
      IF(LOAD.EQ.1) GO TO 60
      IF(LOAD.EQ.2) GO TO 61
60    PAY2(J) = MS - M3 - M4
      PAY2(J) = PAY2(J) * SLUG
      GO TO 62
61    PAY2(J) = PAY * SLUG
62    WRITE(6,302) SIMP,(ROOTR(JK),JK=1,4),PAY2(J)
302   FORMAT(5X,E15.8,2X,4E15.8,2X,E15.8)
      WRITE(6,303)(ROOTI(JK),JK= 1,4)
303   FORMAT(22X,4E15.8)
```

```
      IF(LOAD.EQ.1) MS = M3 + M4
      IF(LOAD.EQ.2) GO TO 9
      MS = MS + .31081 * I
11    CONTINUE
  23  CONTINUE
9     CONTINUE
17    CONTINUE
      END
```

```
      SUBROUTINE COEFF(R1,R2,THETA,XINC1,XINC2,VX1,VY1,VX2,VY2,XIMP,
     1COE)
      COMMON/POLY/W1,W2,W3,W4,W5,ALPHA1,ALPHA2,ALPHA3,ALPHA4,ALPHA5,
     1ALPHA6,ALPHA7,ALPHA8
      COMMON/COF/ BETA1,BETA2,BETA3,BETA4,BETA5,BETA6,BETA7,BETA8
      DOUBLE PRECISION COE(5)
      W1 = (1. - COS(THETA))/SIN(THETA)
      W2 = (R2 * COS(THETA) - R1)/(R1 * R2 * SIN(THETA))
      W3 = 1./R1
      W4 = (R2 - R1 * COS(THETA))/(R1 * R2 * SIN(THETA))
      W5 = 1./R2
      ALPHA1 = (W2**2 + W3**2)
      ALPHA2 =(-2.)*((VX1 * W2) + (VY1 * W3 * COS(XINC1)))
      ALPHA3 = ((-2.) * VX1 * W1)
      ALPHA4 = W1 **2
      ALPHA5 = 2. * W1 * W2 + VX1**2 + VY1**2
      ALPHA6 =(-2.)*((VX2 * W4) + (VY2 * W5 * COS(XINC2)))
      ALPHA7 = 2. * VX2 * W1
      ALPHA8 = VX2**2 + VY2**2 - 2. * W1 * W4
      BETA1 = ALPHA2 - ALPHA6
      BETA2 = ALPHA3 - ALPHA7
      BETA3 = ALPHA5 - ALPHA8
      BETA4 = 2. * ALPHA1
      BETA5 = ALPHA2 + ALPHA6
      BETA6 = ALPHA3 + ALPHA7
      BETA7 = 2. * ALPHA4
      BETA8 = ALPHA5 + ALPHA8
      COE(1) = (BETA1**2) -2. * (XIMP**2) * BETA4
      COE(2) = (2. * BETA1 * BETA3) - 2. * (XIMP**2) * BETA5
      COE(3) = (2. * BETA1 * BETA2) + (BETA3**2) - 2. * (XIMP**2) *
     1BETA8 + (XIMP**4)
      COE(4) = (2. * BETA2 * BETA3) - 2. * (XIMP**2) * BETA6
      COE(5) = (BETA2**2) - 2. * (XIMP**2) * BETA7
      RETURN
      END
```

```
      SUBROUTINE TRAJ(SQRP,R1,XIMP,I)
      COMMON/POLY/W1,W2,W3,W4,W5,ALPHA1,ALPHA2,ALPHA3,ALPHA4,ALPHA5,
     1ALPHA6,ALPHA7,ALPHA8
      COMMON/COF/ BETA1,BETA2,BETA3,BETA4,BETA5,BETA6,BETA7,BETA8
      COMMON/TRAC/VTX1,VTY1,VTX2,VTY2,VTX3,VTY3,EXT,SMAXIS,ZZ
      DIMENSION ZZ(4)
      DUTU = 2.593627354E4
      P = SQRP * SQRP
      VTX1 = W1/SQRP + W2*SQRP
      VTY1 = W3 * SQRP
      VSQ = VTX1 ** 2 + VTY1 ** 2
      VTX2 = -W1/SQRP + W4 * SQRP
      VTY2 = W5 * SQRP
      V2SQ = VTX2 ** 2 + VTY2 ** 2
      VTX1 = VTX1 * DUTU
      VTY1 = VTY1 * DUTU
      VTX2 = VTX2 * DUTU
      VTY2 = VTY2 * DUTU
      ENERGY = (VSQ/2.) - (1./R1)
C           CHECK FOR ELLIPTICAL,PARABOLIC OR HYPERBOLIC TRANSFER ORBIT
      IF(ENERGY) 1,2,3
 1    SMAXIS = -1./(2. * ENERGY)
      EXT = SQRT(1. - (P/SMAXIS))
      GO TO 20
 2    EXT = 1.
      SMAXIS = 1.E19
      GO TO 20
 3    SMAXIS = -1./(2. * ENERGY)
      EXT = SQRT(1. - (P/SMAXIS))
20    CONTINUE
C           THE VELOCITY CHANGE IS FOUND AT EACH IMPULSE
      DV1SQ = (ALPHA1*P) + (ALPHA2*SQRP) + (ALPHA3/SQRP) + (ALPHA4/P)
     1 + ALPHA5
      DV2SQ = (ALPHA1*P) + (ALPHA6*SQRP) + (ALPHA7/SQRP) + (ALPHA4/P)
     1+ ALPHA8
```

```
      DV1 = SQRT(DV1SQ)
      DV2 = SQRT(DV2SQ)
      TIMP = DV1 + DV2
C           CHECK THAT VELOCITY USED IS SAME AS IMPULSE AVAILABLE
      ZZ(I) = ABS(TIMP - XIMP)
      WRITE(6,72) ZZ(I)
72    FORMAT(5X,*IMPULSE CHECK: *,E15.8)
      RETURN
      END
```

```
      PROGRAM XPLOT(INPUT,OUTPUT,TAPE5=INPUT,PLOT)
      DIMENSION X(22),Y(4,22),YPLOT(22)
      INTEGER ORBIT
      READ(5,*) ORBIT,LOAD,N,M
      READ(5,*)  (X(J),J=1,N)
      READ(5,*)  ((Y(I,J),J=1,N),I=1,M)
      CALL  PLOT (0., -4., -3)
      CALL  PLOT (0., 2., -3)
      X(N+1)  =  0.0
      X(N+2)  =  0.7
      YPLOT(N+1)  =  0.0
      YPLOT(N+2)  =  500.
      CALL  AXIS(0., 0., 42HINCLINATION ANGLE FROM INITIAL ORBIT (DEG),
     1-42,5., 0., X(N+1), X(N+2))
      DO 2 J=1,N
      YPLOT(J)  =  Y(1,J)
 2    CONTINUE
      CALL  AXIS(0.,0.,21HMAXIMUM PAYLOAD (LBS), 21,7.,90.,YPLOT(N+1),
     1YPLOT(N+2))
      DO 1 I=1,M
      DO 3 J=1,N
      YPLOT(J)  =  Y(I,J)
 3    CONTINUE
      CALL  FLINE(X,YPLOT,-N,1,1,I)
 1    CONTINUE
      IF(ORBIT.EQ.1.AND.LOAD.EQ.1) GO TO 5
      IF(ORBIT.EQ.1.AND.LOAD.EQ.2) GO TO 7
      IF(ORBIT.EQ.2.AND.LOAD.EQ.1) GO TO 9
      IF(ORBIT.EQ.2.AND.LOAD.EQ.2) GO TO 11
 5    CALL SYMBOL (2.2,5.4, 0.150, 18HFIRST ORBIT CHANGE, 0., 18)
      CALL SYMBOL(2.2,5.2,0.150,17H1ST PAYLOAD KNOWN,0.,17)
      GO TO 11
 7    CALL SYMBOL(2.2,5.4, 0.150, 18HFIRST ORBIT CHANGE,0., 18)
      CALL SYMBOL(2.2,5.2,0.150,17H2ND PAYLOAD KNOWN,0.,17)
      GO TO 11
 9    CALL SYMBOL (2.2,5.4, 0.150, 19HSECOND ORBIT CHANGE, 0., 19)
      CALL SYMBOL(2.2,5.2,0.150,17H1ST PAYLOAD KNOWN,0.,17)
 11   CALL PLOT(5.,0.,-3)
      CALL PLOTE
      END
```

```
      SUBROUTINE ANGLE(XINC1,PHI,A,XINC2,THETA)
      PI = 3.14159265
      XINC2 = ACOS(SIN(XINC1) * SIN(A) * COS(PHI) - COS(XINC1) * COS(A))
      THETA = ASIN(SIN(PHI) * SIN(A)/SIN(XINC2))
      TEST = ((-COS(XINC1)) * SIN(A) * COS(PHI) - SIN(XINC1) * COS(A))/
     1SIN(XINC2)
      IF(TEST) 2,2,1
1     THETA = PI - THETA
2     XINC2 = PI - XINC2
      RETURN
      END

      SUBROUTINE ANGLE1(XINC1,PHI,A,XINC2,THETA,THETA1,THETA2)
      PI = 3.14159265
      XINC2 = PI - XINC2
      THETA1 = PHI
      THETA2 = ASIN(SIN(XINC1) * SIN(PHI)/SIN(XINC2))
      TEST1 = (SIN(PHI) * COS(XINC1))/(SIN(XINC2) * COS(THETA2))
      TEST2 = TEST1 + (SIN(XINC1) * SIN(PHI) * COS(XINC2) * COS(XINC1)
     1* SIN(A)
     1* COS(PHI))/((SIN(XINC2)**3) * COS(THETA2))
      TEST3 = TEST2 +((SIN(XINC1)**2) * SIN(PHI) * COS(XINC2) * COS(A)
     1)/((SIN
     1(XINC2)**3) * COS(THETA2))
      XINC2 = PI - XINC2
      IF(TEST3) 12,10,10
10    IF(XINC1.LT.PI) GO TO 20
      THETA2 = 2. * PI + THETA2
      GO TO 20
12    THETA2 = PI - THETA2
20    RETURN
      END
```

Vita

Rodney Alan Connell was born on 21 November 1947 in Seattle, Washington. He graduated from Englewood High School in Englewood, Colorado, in 1966. He attended the University of Colorado, Boulder, and graduated in 1970 with a Bachelor of Science Degree in Aerospace Engineering. While at the University, he was enrolled in the Air Force ROTC and received a commission in the U. S. Air Force upon graduation. He attended Minuteman missile combat crew training at Vandenberg Air Force Base, California, until April, 1971. He served as a missile launch officer in Minuteman I and Minuteman Modernized - Command Data Buffer missile systems at Francis E. Warren Air Force Base, Wyoming. In January, 1975, he attended Squadron Officer School at Maxwell Air Force Base, Alabama. In June, 1975, he entered the Graduate Astronautical Engineering program at the Air Force Institute of Technology School of Engineering at Wright-Patterson Air Force Base, Ohio.

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: GA/MC/76D-6
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): MAXIMUM PAYLOAD, FOUR-IMPULSE, NON-COPLANAR, ORBITAL TRANSFERS FOR AN UPPER STAGE VEHICLE OF THE SPACE TRANSPORTATION SYSTEM
5. TYPE OF REPORT & PERIOD COVERED: MS Thesis
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Rodney A. Connell, Captain USAF
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Air Force Institute of Technology (AFIT-EN), Wright-Patterson AFB, Ohio 45433
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS
12. REPORT DATE: December, 1976
13. NUMBER OF PAGES: 70
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report): unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES: Approved for public release IAW AFR 190-17

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    Director of Information
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
    - Maximum Payload
    - Impulsive Thrust
    - Orbital Transfer
    - Space Transportation System
    - Upper Stage Vehicle
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Payload capabilities were calculated for an expendable upper stage vehicle compatible with the Space Shuttle Vehicle. Analysis was performed for a four-stage vehicle that was modeled with impulsive thrust and transfer trajectories which obey restricted two-body equations of motion.

The magnitude of the maximum payload deployed into one of two specified orbits when the other payload is known is solved by breaking the four-impulse transfer into two dual-impulse transfer trajectories. → next page

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The maximum payload solution for one transfer depends upon the specified payload of the other transfer. Each of the dual-impulse transfer trajectories is determined by solving a quartic equation in the square root of the semi-latus rectum of the transfer orbit. Maximum payload capability was dependent upon the available impulse, the angle between orbit planes, the difference in the radii of the terminal orbits, the plane changes at departure and arrival points, and the transfer angle. Transfer solutions were programmed on a CDC 6600 digital computer.

Computed results indicate that the model vehicle is capable of many non-coplanar orbit-to-orbit transfers that still yield practical payloads. As the transfer angle deviates from the neighborhood around 180~~°~~ deg and the other geometrical parameters increase, the payload decreases.